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Extracting Decision Rules from Linguistic Data Describing Economic Phenomena. The Approach Based on Decision Systems over Ontological Graphs and PSO

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Abstract

The aim of the paper is to present a heuristic method for extracting the most general decision rules from linguistic data describing economic phenomena included in simple decision systems over ontological graphs. Such decision systems have been proposed to deal with linguistic attribute values, describing objects of interest, which are concepts placed in semantic spaces expressed by means of ontological graphs. Ontological graphs deliver some additional knowledge (the so-called background knowledge) about semantic relations between concepts which can be useful in classification processes. As heuristics, we propose to use Particle Swarm Optimization (PSO), which is reported as a successful method in many applications.

Keywords: decision rules, decision systems, ontological graphs, particle swarm optimization

Introduction

In the majority of cases, data describing economic phenomena have a numeric character. There are a lot of statistical and econometric methods developed for this kind of data. However, in economy, we meet also situations where phenomena are described by linguistic concepts. We assume that linguistic data are included in the so-called decision systems. An important problem concerning decision systems is to extract the knowledge hidden in such systems. This knowledge can be expressed in the form of decision rules. The topic of rule definition and extraction in various decision systems was widely considered in the literature devoted to inductive machine learning (see: Cios et al. 2007; Mitchell 1997). Therefore, we have also considered decision rules in such systems simultaneously with a definition of decision systems over ontological graphs (see: Pancerz 2012a, 2013a). In (Pancerz 2012a), elementary decision rules (i.e., rules having single descriptors on the left-hand sides), consistent with the Dominance-Based Rough Set Approach (Greco, Matarazzo, and Slowiński 2001), have been investigated. In (Pancerz 2013a), elementary decision rules, defined according to different meanings of condition attribute values (exact meaning, synonymous meaning, and general meaning) have been proposed. Now, our investigations are extended to decision rules with complex condition parts (i.e., rules having multi descriptors on the left-hand sides). At the beginning, only a special case is taken into consideration, when descriptors appearing on the left-hand sides of rules are linked by the AND logical connectives.

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In the literature, a variety of taxonomies of different types of semantic relations have been proposed (e.g., Chaffin and Hermann 1988; Milstead 2001; Storey 1993; Winston et al. 1987). Relations are very important components in ontology modeling as they describe the relationships that can be established between concepts. We touch the problem of semantic relations (meaning relations) considered in linguistics. In general, three basic kinds (families) of relations can be distinguished: equivalence relations, hierarchical relations, associative relations (Milstead 2001). For example, synonymy is a kind of an equivalence relation. The generalization ("is-a") relation is the traditional example of hierarchical relations. To deliver the knowledge, called the background knowledge, about semantic relations between concepts, we use ontological graphs which are incorporated into information or decision systems (Pancerz 2012b). Similar approaches using data semantics in information (decision) systems, have been proposed earlier in the literature: Attribute Value Ontology (AVO) (Lukaszewski, Józefowska, and Lawrynowicz 2012), DAG-Decision Systems (Midelfart and Komorowski 2002), Dominance-Based Rough Set Approach (DRSA) (Greco, Matarazzo, and Slowiński 2001), Rough Ontology (Ishizu et al. 2007), etc.

Most common approach to exploit the background knowledge in data mining is generalization of attribute values (concepts) (Han, Cai, and Cercone 1992). It allows to define abstract concepts as generalizations of the primitive ones. Therefore, in the presented approach, we are interested in the generalization relation defined in the ontological graphs. This relation leads us to concept hierarchies representing the necessary background knowledge which controls the generalization process. When attribute values are too specific (concrete, detailed), the discovered knowledge, for example in the form of decision rules, tends to become complicated. Moreover, the quality of rule-based classifiers is dependent, among others, on the levels of abstraction in the data (attribute values) definition. Too specific data (especially noisy data) cause, among others, extracting meaningless knowledge. Moreover, according to Vapnik's statistical learning theory, an important problem is the generalization ability of classifiers (Vapnik 1998). The influence of levels of data abstraction has been considered for different machine learning and data mining problems (e.g., naive Bayesian classification (Łukaszewski et al. 2011)), decision tree induction (Kudoh, Haraguchi, and Okubo 2003; Nunez 1991), mining association rules (Srikant and Agrawal 1997), etc. In (Pancerz 2013b), we have shown how semantic relationships added to decision systems change a look at approximations of sets in the rough set approach.

In the presented paper, we propose a heuristic method for extracting the most general decision rules from linguistic data describing economic phenomena included in simple decision systems over ontological graphs. The level of generalization is determined on the basis of hierarchy fixed by a generalization relation defined by means of ontological graphs associated with attributes in decision systems.

1 Simple decision systems over ontological graphs

In this section, we recall necessary definitions, notions and notation concerning simple decision systems over ontological graphs provided in our earlier papers (see: Pancerz 2012a, 2012b, 2013a, 2013b). Information systems (decision systems) are considered from the Pawlak's perspective, as the knowledge representation systems (see: Pawlak 1991).

Definition 1. A decision system DS is a tuple $DS = (U, C, D, V_c, V_d, c, d)$, where:
U is a nonempty, finite set of objects,
C is a nonempty, finite set of condition attributes,
D is a nonempty, finite set of decision attributes,
$V_c = \bigcup_{a \in C} V_a$, where V_a is a set of values of the condition attribute a,
$V_d = \bigcup_{a \in D} V_a$, where V_a is a set of values of the decision attribute a,
$c : C \times U \to V_c$ is an information function such that $c(a, u) \in V_a$ for each $a \in C$ and $u \in U$,
$d : D \times U \to V_d$ is a decision function such that $d(a, u) \in V_a$ for each $a \in D$ and $u \in U$.

In (Pancerz 2013a), we have proposed to consider attribute values describing objects in the ontological space, where ontology is constructed on the basis of a controlled vocabulary and the relationships of the concepts in the controlled vocabulary (see definitions given by Neches et al. (1991) and Kohler et al. (2006)). In that approach, we use formal representations of ontologies by means of graph structures. Such structures are called ontological graphs. For a given ontology $\mathcal{O}$, an ontological graph includes nodes representing concepts from $\mathcal{O}$ and edges representing relations between concepts from $\mathcal{O}$.

Definition 2. Let $\mathcal{O}$ be a given ontology. An ontological graph is a quadruple $OG = (\mathcal{C}, E, \mathcal{R}, \rho)$, where:
$\mathcal{C}$ is a nonempty, finite set of nodes representing concepts in the ontology $\mathcal{O}$,
$E \subseteq \mathcal{C} \times \mathcal{C}$ is a finite set of edges representing relations between concepts from $\mathcal{C}$,
$\mathcal{R}$ is a family of semantic descriptions (in natural language) of types of relations (represented by edges) between concepts,
$\rho : E \to \mathcal{R}$ is a function assigning a semantic description of the relation to each edge.

In the proposed approach, we take into consideration the following family of semantic descriptions of relations between concepts: "is synonymous with", "is generalized by", "is specialized by" (Pancerz 2012b). Some other relations are also considered (e.g., Pancerz 2012b, 2013b). We will use the following notation: $R_\sim$ "is synonymous with," $R_\triangleleft$ "is generalized by," $R_\triangleright$ "is specialized by." Therefore, for simplicity $\mathcal{R} = \{R_\sim, R_\triangleleft, R_\triangleright\}$.

The relations mentioned have the following properties: $R_\sim$ is reflexive, symmetric and transitive, $R_\triangleleft$ is reflexive and transitive, $R_\triangleright$ is reflexive and transitive. In the graphical representation of the ontological graph, for readability, we will omit reflexivity of relations. However, the above relations are reflexive (i.e., a given concept is synonymous with itself, a given concept is generalized by itself, a given concept is specialized by itself).

We can create decision systems over the ontological graphs. It can be done in different ways. In our investigation, condition attribute values of a given decision system are concepts from ontologies assigned to attributes. Such a system is called a simple decision system over ontological graphs (see: Pancerz 2012b).

Definition 3. A simple decision system SDS^{OG} over ontological graphs is a tuple
$SDS^{OG} = (U, C, D, \{OG_a\}_{a \in C}, V_d, c, d)$, where:
U is a nonempty, finite set of objects,
C is a nonempty, finite set of condition attributes,
D is a nonempty, finite set of decision attributes,
$\{OG_a\}_{a \in C}$ is a family of ontological graphs associated with condition attributes from C,
$V_d = \bigcup_{a \in D} V_a$ is a set of values of the decision attribute $a \in D$,
$c : C \times U \to \mathcal{C}$, $\mathcal{C} = \bigcup_{a \in C} \mathcal{C}_a$ is an information function such that $c(a, u) \in \mathcal{C}_a$ for each $a \in C$ and $u \in U$, $\mathcal{C}_a$ is a set of concepts from the graph OG_a,
$d : D \times U \to V_d$ is a decision function such that $d(a, u) \in V_d$ for each $a \in D$ and $u \in U$.

Remark 1. It is not necessary for an information function to be a total function
(i.e., $c : U \times C \to \mathcal{C}^* \subseteq \mathcal{C}$).

In our approach, we propose to consider some relations defined over sets of attribute values in simple decision systems over ontological graphs. In the relations defined above we use some additional knowledge about relationships between attribute values which is included in ontological graphs.

Let $OG = (\mathcal{C}, E, \mathcal{R}, \rho)$ be an ontological graph. We will use the following notation: $[c_i, c_j]$ is a simple path in OG between $c_i, c_j \in \mathcal{C}$, $\mathcal{E}([c_i, c_j])$ is a set of edges from E belonging to the simple path $[c_i, c_j]$, $\mathcal{P}(OG)$ is a set of all simple paths in OG. In the literature, there are different definitions for a simple path in the graph. In this paper, we follow the definition in which a path is simple if no node or edge is repeated, with the possible exception that the first node is the same as the last. Therefore, the path $[c_i, c_j]$ where $c_i, c_j \in \mathcal{C}$ and $c_i = c_j$ can also be a simple path in OG.

Definition 4. Let an ontological graph $OG_a = (\mathcal{C}_a, E_a, \mathcal{R}, \rho_a)$ be associated with the attribute a in a simple decision system over ontological graphs, where $\mathcal{R} = \{R_\sim, R_\lhd, R_\rhd\}$.
An exact meaning relation between $c_1, c_2 \in \mathcal{C}_a$ is defined as

(1) $$EMR(a) = \{(c_1, c_2) \in \mathcal{C}_a \times \mathcal{C}_a : c_1 = c_2\}.$$

A synonym meaning relation between $c_1, c_2 \in \mathcal{C}_a$ is defined as

(2) $$SMR(a) = \left\{(c_1, c_2) \in \mathcal{C}_a \times \mathcal{C}_a : \underset{[c_1,c_2]\in\mathcal{P}(OG_a)}{\exists} \; \underset{e\in\mathcal{E}([c_1,c_2])}{\forall} \rho_a(e) = R_\sim\right\}.$$

A generalization relation $GR(a)$ between $c_1, c_2 \in \mathcal{C}_a$ is defined as

(3) $$GR(a) = \left\{(c_1, c_2) \in \mathcal{C}_a \times \mathcal{C}_a : \underset{[c_1,c_2]\in\mathcal{P}(OG_a)}{\exists} \; \underset{e\in\mathcal{E}([c_1,c_2])}{\forall} \rho_a(e) \in \{R_\sim, R_\lhd\}\right\}.$$

A specialization relation $SR(a)$ between $c_1, c_2 \in \mathcal{C}_a$ is defined as

(4) $$SR(a) = \left\{(c_1, c_2) \in \mathcal{C}_a \times \mathcal{C}_a : \underset{[c_1,c_2]\in\mathcal{P}(OG_a)}{\exists} \; \underset{e\in\mathcal{E}([c_1,c_2])}{\forall} \rho_a(e) \in \{R_\sim, R_\rhd\}\right\}.$$

If $(c_1, c_2) \in SMR(a)$ then c_1 and c_2 are synonyms. If $(c_1, c_2) \in GR(a)$ then c_1 is generalized by c_2. If $(c_1, c_2) \in SR(a)$ then c_1 is specialized by c_2.

2 Decision rules in simple decision systems over ontological graphs

Decision rules in simple decision systems over ontological graphs have been considered in (Pancerz 2013a) (in accordance with different meanings of condition attribute values defined with respect to relations between concepts) as well as in (Pancerz 2012a) (in accordance to Dominance-Based Rough Set Approach (DRSA) (Greco, Matarazzo, and Słowiński 2001)). Here, we recall some concepts proposed in (Pancerz 2013a).

Let $SDS^{OG} = (U, C, D, \{OG_a\}_{a\in C}, V_d, c, d)$ be a simple decision system over ontological graphs. Let $\mathcal{C} = \bigcup_{a\in C} \mathcal{C}_a$, where $\mathcal{C}_a$ is a set of concepts from the graph OG_a associated with a given $a \in C$. For the decision system SDS^{OG}, we define condition descriptors which are expressions (a, v) over C and $\mathcal{C}$, where $a \in C$ and $v \in \mathcal{C}$ as well as decision descriptors which are expressions (a, v) over D and V_d, where $a \in D$ and $v \in V_d$.

In a given simple decision system $SDS^{OG} = (U, C, D, \{OG_a\}_{a\in C}, V_d, c, d)$ over ontological graphs, we will consider decision rules in the form

(5) $$(a_{c_1}, v_{c_1}) \wedge (a_{c_2}, v_{c_2}) \wedge \cdots \wedge (a_{c_k}, v_{c_k}) \Rightarrow (a_d, v_d)$$

where $a_{c_1}, a_{c_2}, \ldots, a_{c_k} \in C$, $v_{c_i} \in \mathcal{C}_{a_{c_i}}$ of $OG_{a_{c_i}}$, $i = 1, 2, \ldots, k$, $a_d \in D$, $v_d \in V_d$.

Let $U_C(a, v) = \{u \in U : (c(a, u), v) \in SR(a)$ or $(c(a, u), v) \in SMR(a)\}$, where $a \in C$, and $U_D(a, v) = \{u \in U : d(a, u) = v\}$, where $a \in D$. Decision Rule 2 is true in SDS^{OG} if and only if

(6) $$\left(\bigcap_{i=1,2,\ldots,k} U_C(a_{c_i}, v_{c_i})\right) \subseteq U_D(a_d, v_d)$$

and

(7) $$\left(\bigcap_{i=1,2,\ldots,k} U_C(a_{c_i}, v_{c_i})\right) \neq \emptyset.$$

In the presented approach, we are intersted in the most general decision rules true in a given simple decision system over ontological graphs. Decision Rule 2 is said to be the most general rule with respect to its condition part for a fixed decision part (a_d, v_d) if and only if the rule is true in SDS^{OG} and the rule

(8) $$(a_{c_1}, v'_{c_1}) \wedge (a_{c_2}, v'_{c_2}) \wedge \cdots \wedge (a_{c_k}, v'_{c_k}) \Rightarrow (a_d, v_d),$$

where $(v_{c_i}, v'_{c_i}) \in GR(a_{c_i})$ but $(v_{c_i}, v'_{c_i}) \notin EMR(a_{c_i})$ according to $OG_{a_{c_i}}$, for any $i = 1, 2, \ldots, k$, is not true in SDS^{OG}.

3 PSO based searching for the most general decision rules

In this section, we describe the main idea of searching for the most general decision rules in simple decision systems over ontological graphs. The presented approach uses heuristics in the form of Particle Swarm Optimization (PSO). PSO is reported as a successful method in many applications (Poli 2008; Poli, Kennedy, and Blackwell 2007). In PSO, a number of simple entities (the particles) are placed in the search space of some problem or function, and each evaluates the objective function at its current location.

In our problem, the particles are placed in the space consisting of concepts from the ontological graphs associated with condition attributes in simple decision systems over ontological graphs.

Let $SDS^{OG} = (U, C, D, \{OG_a\}_{a \in C}, V_d, c, d)$, be a simple decision system over ontological graphs, where $C = \{a_1, a_2, \ldots, a_m\}$ and $OG_a = (\mathcal{C}_a, E_a, \mathcal{R}, \rho_a)$ for $a \in C$, . Each individual in the particle swarm is described by three m-dimensional vectors: the current position, the previous best position, and the velocity. Therefore, the search space $\Pi = \mathcal{C}_{a_1} \times \mathcal{C}_{a_2} \times \cdots \times \mathcal{C}_{a_m}$ and the position of the individual $\mathbf{c} = [v_{c_1}, v_{c_2}, \ldots, v_{c_m}]$, where $v_{c_i} \in \mathcal{C}_{a_i}$, for $i = 1, 2, \ldots, m$. The best position found so far is determined on the basis of the fitness function taking into consideration levels at which concepts $v_{c_1}, v_{c_2}, \ldots, v_{c_m}$ are placed in ontological graphs according to a generalization relation. The fitness function

(9) $$\chi(\mathbf{c}) = \sum_{i=1}^{m} \delta(v_{c_i}),$$

where $v_{c_i} \in \mathcal{C}_{a_i}$ in OG_{a_i}, $i = 1, 2, \ldots, m$, and $\delta(v_{c_i}) = 0$ if v_{c_i} is a root concept, $\delta(v_{c_i}) = 1$ if v_{c_i} is placed at the first level from the root, $\delta(v_{c_i}) = 2$ if v_{c_i} is placed at the second level from the root, etc. In our approach, the fitness function is minimized.

For each particle, its movement through the search space is determined by combining some aspect of the history of its own current and best positions with those of one or more members of the swarm, with some random perturbations. The velocity vector shows numbers of levels that we are moving (up or down) in the concept hierarchies determined by generalization relations in ontological graphs associated with condition attributes. The movement comes into effect if a new position $\mathbf{c}' = [v'_{c_1}, v'_{c_2}, \ldots, v'_{c_m}]$ determines a rule

(10) $$(a_1, v'_{c_1}) \wedge (a_2, v'_{c_2}) \wedge \cdots \wedge (a_m, v'_{c_m}) \Rightarrow (a_d, v_d)$$

true in SDS^{OG}.

It is worth noting that if $\delta(v_{c_i}) = 0$ for some $i = 1, 2, \ldots, m$, then the concept v_{c_i} appearing in the rule is a root concept. It means that the atomic formula (a_i, v_{c_i}) can be removed from the rule, because the attribute a_i is dispensable.

Example 1. Let $SDS^{OG} = (U, C, D, \{OG_a\}_{a \in C}, V_d, c, d)$ be a simple decision system describing some economic phenomena, represented by a decision table (see tab. 1), over ontological graphs shown in figure 1. $C = \{Sector, Region\}$ is a set of condition attributes. $D = \{Level\}$ is a set of decision attributes.

Let us fix a decision part as (*Level*, *Medium*). If one of the individuals in the particle swarm is described by a position vector $\mathbf{c}$ = [*Insurance*, *Latin America*], then δ(*Insurance*) = 3 and δ(*Latin America*) = 2, hence χ(c) = 5. The position vector $\mathbf{c}$ determines a rule ρ

(11) $$(Sector, Insurance) \wedge (Region, Latin\ America) \Rightarrow (Level, Medium).$$

The rule ρ is true in SDS^{OG}. Performing the PSO algorithm, we can obtain that the individual is moved in the search space into a new position **c'** = [*Economy Sector*, *Latin America*]. For this position, $\delta(Economy\ Sector) = 0$ and $\delta(Latin\ America) = 2$, hence $\chi(c) = 2$. The attribute *Sector* is dispensable, because of $\delta(Economy\ Sector) = 0$. The position vector **c'** determines a rule ρ'

$$(Region, Latin\ America) \Rightarrow (Level, Medium). \quad (12)$$

The rule ρ' is also true in SDS^{OG} but it is more general than the rule ρ.

Tab. 1. A simple decision system over ontological graphs describing some economic phenomena

U/C ∪ D	Sector	Region	Level
u_1	Forestry	Northern America	High
u_2	Forestry	Caribbean	Medium
u_3	Forestry	Latin America	Medium
u_4	Forestry	Middle East	Low
u_5	Mining	Middle East	High
u_6	Financial Services	Northern America	High
u_7	Legal Services	Northern America	High
u_8	Insurance	Northern America	High
u_9	Financial Services	Latin America	Medium
u_0	Legal Services	Latin America	Medium
u_{11}	Insurance	Latin America	Medium
u_{12}	Industry	Far East	High
u_{13}	Industry	Asia Pacific	Medium
u_{14}	Industry	Middle East	Medium
u_{15}	Mining	Far East	Low

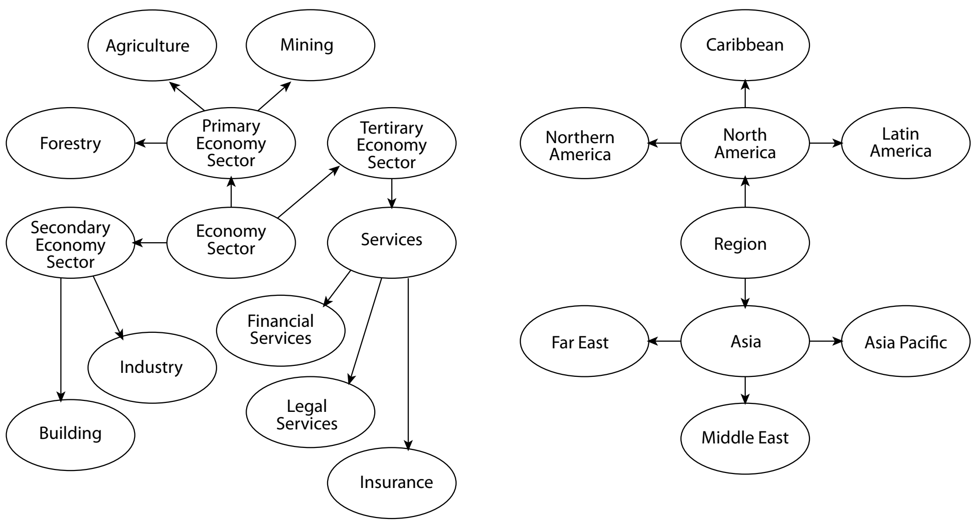

Fig. 1. Ontological graphs OG_{Sector} and OG_{Region} associated with the attributes *Sector* and *Region*, respectively

Conclusions and further works

In the paper, we have shown the main idea of searching for the most general decision rules in the linguistic data describing economic phenomena included in simple decision systems over ontological graphs. The searching space consists of concepts from the ontological graphs associated with condition attributes in such decision systems. The approach is useful if the decision system includes a huge number of condition attributes and the brute force method is ineffective. In the future, we plan to apply the proposed approach for extraction of rules consistent with the Dominance-Based Rough Set Approach in simple decision systems over ontological graphs. Moreover, our attention will be focused on more exact development of the PSO procedure.

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