

# Modeling of Passenger Rail Transport in the Zamość Region Using Multi-Instancely Marked Petri Nets over Ontological Graphs

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## Abstract

*The article discusses the possibility of using Petri nets over ontological graphs to model passenger rail transport in the Zamość region. Petri nets are a mathematical and graphical tool for modeling the structure and dynamics of systems. The Petri net model, proposed for use, also takes into account the semantics of the system elements (e.g., the classes of trains run by carriers). The structure of the passenger rail transport system includes primarily railway lines and passenger rail infrastructure facilities (railway stations and passenger stops). The dynamics of the passenger rail transport system is the movement of trains on railway lines. The approach proposed in the article adopts a higher level of modeling abstraction, which can be made more detailed in further research, primarily by taking into account time dependencies.*

**Keywords:** modelling, Petri nets, OWL ontology, passenger railway transport, the Zamość region

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## Introduction

Modeling of passenger rail transport can address various aspects, for example:

- forecasting the number of passengers that is of key importance for planning the frequency of rail transport (Borucka and Guzanek 2022),
- assessment of transport services that is of key importance for maintaining a proper level of customer satisfaction (Borucka 2020),
- assessment of capacity consumption in railway networks that is of key importance for rail transport management (Mussone and Wolfler Calvo 2013), or
- assessment of the operating quality of timetables that is of key importance for planning rail transport (Büker and Seybold 2012).

All mentioned aspects are modeled using quantitative approaches. These approaches make it possible to assess or predict specific parameters describing passenger rail transport. In the article, we address a slightly different aspect of modeling, namely, our interest is in a comprehensive look at the structure and dynamics of the passenger rail transport system in a defined region (i.e., the Zamość region). First of all, in our approach, we use a tool that, in addition to having a solid mathematical foundation, allows us to graphically present the model. This tool are Petri nets, introduced in the 1960s as a tool for modeling concurrent processes in computer science (Petri 1962). Petri nets offer graphical notation for stepwise processes, like UML activity diagrams and BPMN (Business Process Model and Notation) diagrams. However, Petri nets have also an exact mathematical definition of their execution semantics, with a well-developed mathematical theory for process analysis. Thanks

to this, we can use formal tools to analyze the behavior of systems and verify their correctness (Girault and Valk 2003). Over time, Petri nets have become one of the tools for modeling systems in various areas, including industry (Zurawski and Zhou 1994). Over time, various types of Petri nets differing in structure, dynamics and applications have also been proposed (Best and Devillers 2024; David and Alla 2010). Moreover, the proposed Petri net model takes into account the semantics of the system elements (Szkola and Pancerz 2017). This semantics is expressed by ontological graphs assigned to the model. Ontological graphs are obtained from ontologies built in accordance with the OWL 2 Web Ontology Language standardized by W3C Consortium.<sup>1</sup> Ontologies specify, among others, concepts and the relations between them in individual areas of life, for example in transport systems as it is shown in the paper. It is worth noting that Petri nets over ontological graphs combine the power of graphical and formal description of the structure and dynamics of systems with operations on linguistic data in the form of concepts and relations between them.

In the article, we propose to extend instancely marked Petri nets over ontological graphs, considered, among others, in (Biernacka and Pancerz 2024) to multi-instancely marked Petri nets over ontological graphs. This feature is very important from the point of view of real-life systems, for example, if more trains (represented by train instances) stop at a given station at the same time. Moreover, this feature enables us to use some data structures (e.g., queues, stacks, etc., to store instances).

## 1 Ontologies and Ontological Graphs

Ontologies specify the concepts and the relationships among them in a given domain of interest (Neches et al. 1991). In our approach, we have selected the OWL 2 Web Ontology Language (shortly OWL 2) that is very popular in ontological engineering to implement ontology, and next to obtain ontological graphs. The OWL ontology consists of three components:

- classes (representations of concepts from the domain of interest, interpreted as sets that contain individuals),
- individuals (instances of classes that represent objects in the domain of interest),
- properties (binary relations on individuals) of two types:
  - object properties linking an individual to an individual, and
  - data properties linking an individual to a data value.

Our domain of interest is a passenger rail transport. This domain has been narrowed down to the Zamość region to show the idea of modeling transport system in a simple way. There is nothing stopping us from extending this field to passenger transport system throughout Poland.

Formally, a given ontology  $O$  can be partially represented by means of graph structures called ontological graphs (Pancerz 2012). The ontological graph is a tuple including:  $C$ —the finite set of nodes representing concepts or individuals in the ontology  $O$ ,  $E$ —the finite set of edges representing semantic relations between concepts or individuals,  $R$ —the family of semantic descriptions (in a natural language) of types of relations between concepts or individuals (represented by edges),  $\rho$ —the function assigning a semantic description of the relation to each edge.

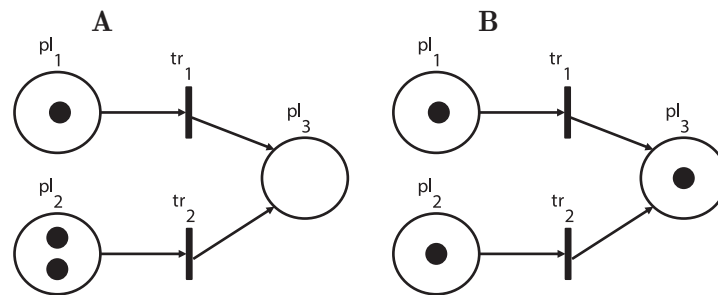
## 2 Multi-Instancely Marked Petri Nets over Ontological Graphs

Petri nets proposed in (Petri 1962), and next developed by many researchers, are now a powerful graphical and formal tool used to describe structures and dynamics of real-life systems. In general, in Petri nets, each place (corresponding to a state of a system, drawn with circles) contains a dynamically varying number of tokens of type depending on a type of Petri net models (see table 1). Therefore, tokens can have different interpretations. An arbitrary distribution of tokens on the places is called a marking. The Petri net dynamics is given by firing enabled transitions (drawn with rectangles) causing the movement of tokens through the net.

1. See: “OWL 2 Web Ontology Language. Structural Specification and Functional-Style Syntax (Second Edition)” by Boris Motik, Peter F. Patel-Schneider, and Bijan Parsia. W3C Recommendation 11 December 2012, available at <https://www.w3.org/TR/owl2-syntax/>.

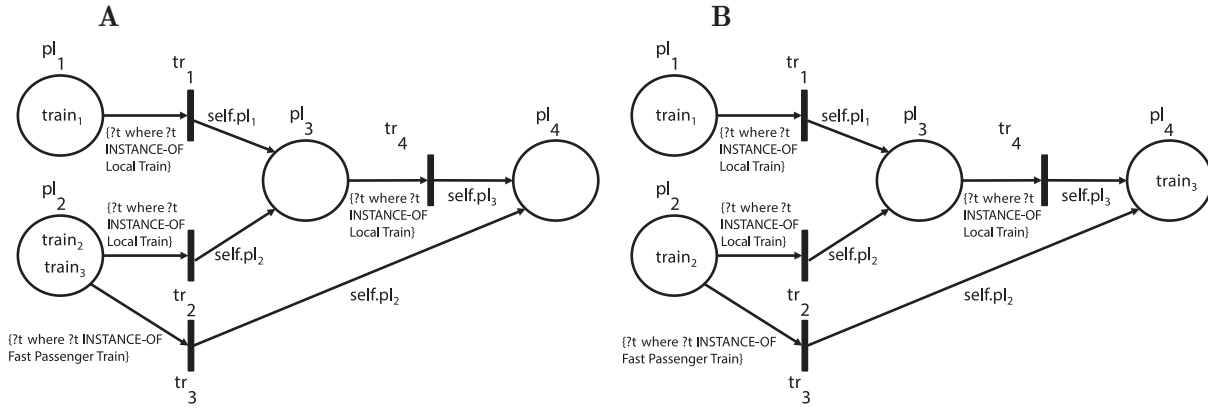
**Table 1.** Selected Petri net models and tokens in their places

Petri Net Model	Tokens in Places	Reference
Place-Transition (P/T) nets	Indiscernible tokens	(Reisig 1985)
Fuzzy Petri nets	Real values between zero and one	(Looney 1994)
Coloured Petri Nets	Multisets over colour sets associated with places	(Jensen and Kristensen 2009)
Predicate-Transition (Pr/T) nets	Formal sums of n-tuples of individuals (items)	(Genrich and Lautenbach 1981)
OBJSA nets	Tokens of abstract data types defined using the language OBJ2	(Battiston, De Cindio, and Mauri 1991)
Conceptually marked Petri nets over ontological graphs	Concepts from ontological graphs associated with places	(Pancerz, Grochowalski, and Derkacz 2016)
Instantly marked Petri nets over ontological graphs	Instances of concepts from ontological graphs associated with places	(Pancerz, Grochowalski, and Paja 2017)

**Figure 1.** A simple Petri net model in the form of a Place-Transition (P/T) net. A, Initial marking (state). B, Marking (state) after firing transition  $tr_2$ .

Let us consider Place-Transition (P/T) nets. A simple Petri net model can have a form as in figure 1A. Let us assume that, in this model, places,  $pl_1$ ,  $pl_2$ , and  $pl_3$  (drawn with circles) represent railway stations. Tokens (drawn with black dots inside places) represent trains at railway stations. Firing transition  $tr_2$  represents the train's passage from the station represented by  $pl_2$  to the station represented by  $pl_3$ . It is worth noting that just as tokens in the model are indistinguishable, trains are also indistinguishable. In this case, we only have information that a train has passed from one station to another. This Petri net model can be treated as a low-level model. Therefore, in (Pancerz, Grochowalski, and Paja 2017; Szkoła and Pancerz 2017), we have proposed the Petri net model, called Petri nets over ontological graphs, in which the domain knowledge is enclosed in a form of ontologies. This Petri net model enables us to obtain much more succinct and expressive descriptions than can be obtained by means of low-level Petri nets (compare figures 1 and 2). In this model, we intend to consider tokens marking Petri nets as entities placed in semantic spaces represented by ontologies. Petri nets over ontological graphs can be treated as the so-called high-level Petri nets. In (Pancerz, Grochowalski, and Paja 2017), we have distinguished two types of Petri nets over ontological graphs: conceptually marked Petri nets over ontological graphs and instantaneously marked Petri nets over ontological graphs (see table 1). In the second case, each place of a model can store at most one instance which is a limitation in the expressive power of models of real systems. In the example of modeling a passenger rail system, there could only be one train at a given station. Therefore, we propose to extend instantaneously marked Petri nets over ontological graphs, considered earlier, to multi-instantaneously marked Petri nets over ontological graphs. In this model, instances are stored in a given place according to a selected data structure (set, list, map, stack, or queue) for this place (see sketch of a formal definition in table 2).

Let us again model a simple passenger rail transport system similar to the previous one. A model in the form of a multi-instantaneously marked Petri net over ontological graphs is shown in figure 2 on next page.

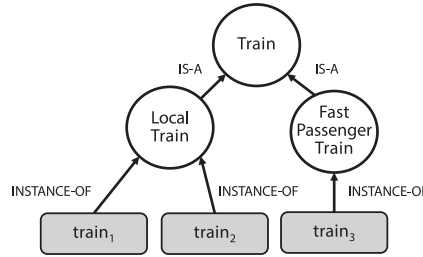


**Figure 2.** A simple Petri net model in the form of a multi-instancely marked Petri net over ontological graphs. A, Initial marking (state). B, Marking (state) after firing transition  $tr_3$ .

An ontological graph assigned to all places ( $pl_1$ ,  $pl_2$ ,  $pl_3$ , and  $pl_4$ ) in the model shown in figure 2 is drawn in figure 3. One can see two basic semantic relations, namely:

- IS-A (if  $c$  IS-A  $c'$ , then  $c$  is a kind of  $c'$ ), and
- INSTANCE-OF (if  $i$  INSTANCE-OF  $c$ , then  $i$  is an instance of  $c$ ).

Tokens (trains) are placed in the semantic space determining their types (either a local train or a fast passenger train). Taking this knowledge into consideration, we can see that transition  $tr_3$  can be fired (see figure 2B). Transition  $tr_3$  can take from place  $pl_2$  only  $train_3$ , because its input arc formula (i.e.,  $\{?t \text{ where } ?t \text{ INSTANCE-OF Fast Passenger Train}\}$ ) says that the train (variable  $?t$ )



**Figure 3.** An ontological graph assigned to all places in the model shown in figure 2

**Table 2.** Items of a multi-instancely marked Petri net over ontological graphs

Item	Description	Remarks
$Pl$	The finite set of places	
$Tr$	The fine set of transitions	
$\{OG\}_{p \in Pl}$	The family of ontological graphs associated with places	Ontological graphs from $\{OG\}_{p \in Pl}$ define semantic spaces for tokens distributed over places
$DS_{Pl}$	The place data structure function	$DS_{Pl}$ assigns a data structure (set, list, map, stack, or queue) to each place
$Arc_{in}$	The set of input arcs	$Arc_{in}$ is a subset of $Pl \times Tr$
$Arc_{out}$	The set of output arcs	$Arc_{out}$ is a subset of $Tr \times Pl$
$Form_{in}$	The input arc formula function	$Form_{in}$ assigns a formula (whose value is a subset of the set of instances from the ontological graph assigned to the place) to each input arc
$Form_{out}$	The output arc formula function	$Form_{out}$ assigns a formula (whose value is an instance from the ontological graph assigned to the place) to each output arc
$Mark_0$	The initial marking function	$Mark_0$ determines an initial state (i.e., distribution of tokens over places)

can be only a fast passenger train. In this way, we model that  $train_3$  goes directly to the station represented by place  $pl_4$ . Trains  $train_1$  and  $train_2$  are local trains. Hence, transitions  $tr_1$  and  $tr_2$  can be fired, respectively. Their input arc formulas are  $\{?t \text{ where } ?t \text{ INSTANCE-OF Local Train}\}$ . In this way, we model that  $train_1$  and  $train_2$  go to the station represented by place  $pl_3$ . The output arc formulas consist of the special variable  $?self.place$  representing a situation that the same token taken from an input place is put to an output place. We are developing PNOGDL (Petri Nets over Ontological Graphs Description Language) that is the formal language to describe entities of models (e.g., arc formulas).

### 3 Passenger Rail Transport in the Zamość Region

The Zamość region is a region with very little railway lines. According to the list of PKP Polskie Linie Kolejowe S.A., there are three railway lines on which passenger trains run: 66 (see table 3), 69 (see table 4) and 72 (see table 5). The tables also present information on the number of platforms and tracks next to them. This information is used to build the model. We can distinguish two types of passenger railway facilities: railway stations and passenger stops. At the passenger stop (due to the lack of available infrastructure), unlike at the railway stations, trains cannot pass (i.e., cross) or overtake each other. This distinction is taken into account in the model.

**Table 3.** Railway line 66 (Zwierzyniec Towarowy – Stalowa Wola Południe) provided by PKP Polskie Linie Kolejowe S.A. in the Zamość Region

Passenger Railway Facility	Type	Number of platforms	Number of tracks <sup>a</sup>
Zwierzyniec Towarowy	Railway Station	—	—
Tereszpol Biłgorajski	Passenger Stop	1	1
Biłgoraj	Railway Station	1	2

<sup>a</sup> Only tracks at platforms are included.

**Table 4.** Railway line 69 (Rejowiec – Hrebenne) provided by PKP Polskie Linie Kolejowe S.A. in the Zamość Region

Passenger Railway Facility	Type	Number of platforms	Number of tracks <sup>a</sup>
Rejowiec	Railway Station	2	3
Zagrody Kościół	Passenger Stop	1	1
Żulin	Passenger Stop	1	1
Krasnystaw Fabryczny	Railway Station	1	2
Krasnystaw Miasto	Passenger Stop	1	1
Wólka Orłowska	Passenger Stop	1	1
Izbica	Passenger Stop	1	1
Tarzymiechy	Passenger Stop	1	1
Krzak	Passenger Stop	1	1
Złojec	Passenger Stop	1	1
Zawada	Railway Station	1	2
Wólka Niedzieliska	Passenger Stop	1	1
Niedzieliska-Kolonia	Passenger Stop	1	1
Szczebrzeszyn	Passenger Stop	1	1
Żurawnica	Passenger Stop	1	1
Zwierzyniec	Passenger Stop	1	1
Józefów Roztoczański	Passenger Stop	1	1

<sup>a</sup> Only tracks at platforms are included.

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Tab.4 (continued)

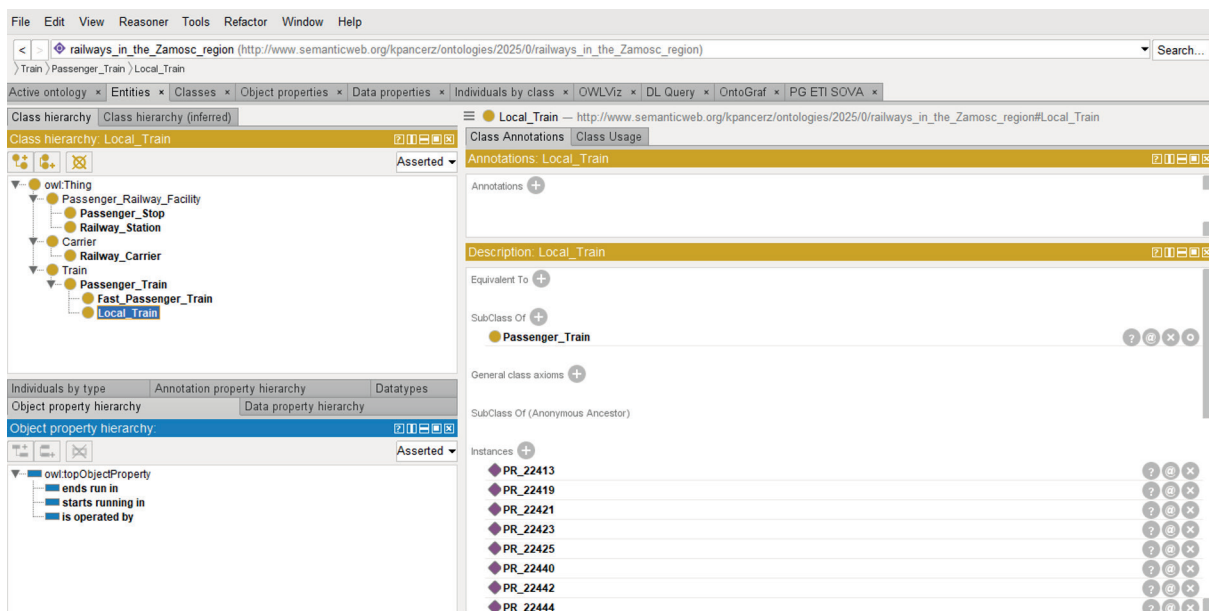
Passenger Railway Facility	Type	Number of platforms	Number of tracks <sup>a</sup>
Długi Kąt	Passenger Stop	1	1
Nowiny	Passenger Stop	1	1
Susiec	Railway Station	2	2
Maziły	Passenger Stop	1	1
Bełzec Drugi	Passenger Stop	1	1
Bełzec	Railway Station	3	3
Lubycza Królewska	Passenger Stop	1	1
Hrebenne	Railway Station	1	2

<sup>a</sup> Only tracks at platforms are included.**Table 5.** Railway line 72 (Zawada – Hrubieszów Miasto) provided by PKP Polskie Linie Kolejowe S.A. in the Zamość Region

Passenger Railway Facility	Type	Number of platforms	Number of tracks <sup>a</sup>
Zawada	Railway Station	1	2
Mokre	Passenger Stop	1	1
Zamość	Railway Station	1	2
Zamość Starówka	Passenger Stop	1	1
Zamość Wschód	Passenger Stop	1	1
Werbkowice	Railway Station	1	2
Hrubieszów Miasto	Railway Station	2	2

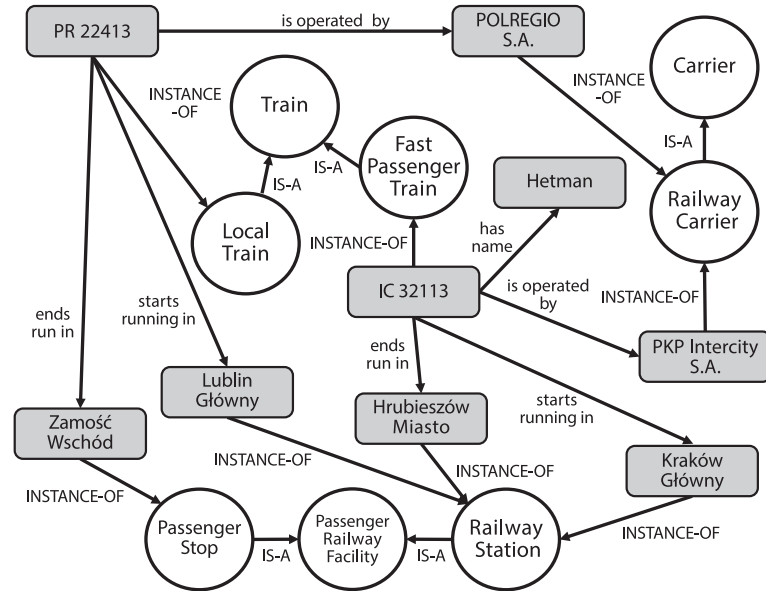
<sup>a</sup> Only tracks at platforms are included.

The OWL2 ontology to deliver ontological graphs for the model of passenger rail transport in the Zamość region, in the form of a multi-instancely marked Petri net, has been created using the Protege environment (Musen 2015) shown in figure 4. One can see the hierarchy of classes constituted by the IS-A semantic relation. We have distinguished three top-level classes: Carrier, Passenger Railway Facility, Train.

**Figure 4.** The fragment of the OWL2 ontology for modeling of passenger rail transport in the Zamość region in the Protege environment



The fragment of the ontological graph derived from the ontology for modeling of passenger rail transport in the Zamość region is shown in figure 5. One can see classes (*Train*, *Fast Passenger Train*, *Local Train*, *Carrier*, *Railway Carrier*, *Passenger Railway Facility*, *Railway Station*, *Passenger Stop*), individuals (*IC 32113*, *PR 22413*, *POLREGIO S.A.*, *PKP Intercity S.A.*, *Hrubieszów Miasto*, *Kraków Główny*, *Lublin Główny*, *Zamość Wschód*), object properties (is operated by, starts running in, ends run in) as well as a data property (has name).



**Figure 5.** The fragment of the ontological graph derived from the ontology for modeling of passenger rail transport in the Zamość region

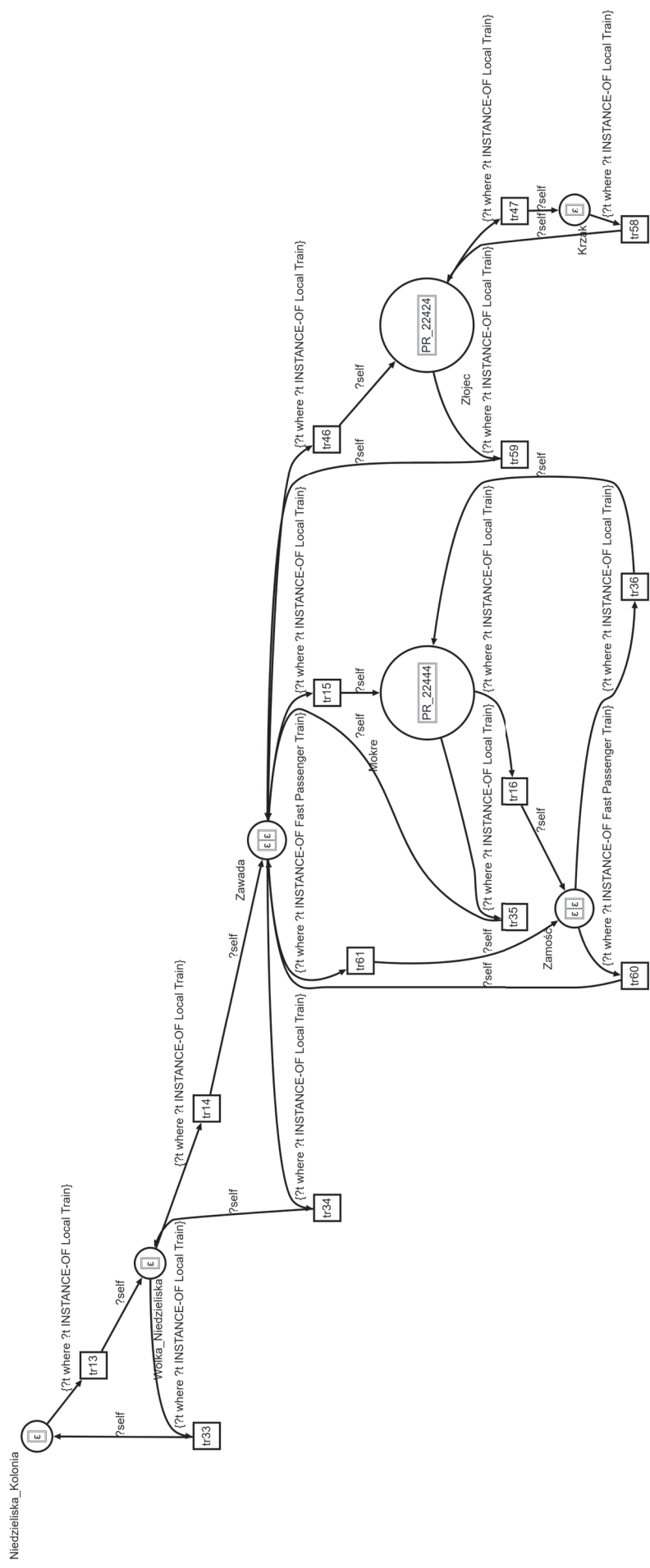
The fragments of the multi-instancely marked Petri net over ontological graphs modeling of passenger rail transport in the Zamość region are shown in figures 6 (an initial marking) and 7 (a marking after firing transitions *tr59* and *tr35*; on next page). One can see that trains from Złojec and Mokre pass each other in Zawada. Each place has a data structure of a fixed size assigned to it. In the presented net, it is a list. The size of the list is equal to the number of tracks at platforms available at a railway station or a passenger stop represented by the place (see tables 3, 4, and 5). Input arc formulas determine which tokens can be taken by connected transitions. One can see that instances at different levels of abstraction are included.

## Conclusions

Petri nets over ontological graphs can be used wherever linguistic concepts are used. The approach presented in the paper fits into several important areas of modern artificial intelligence:

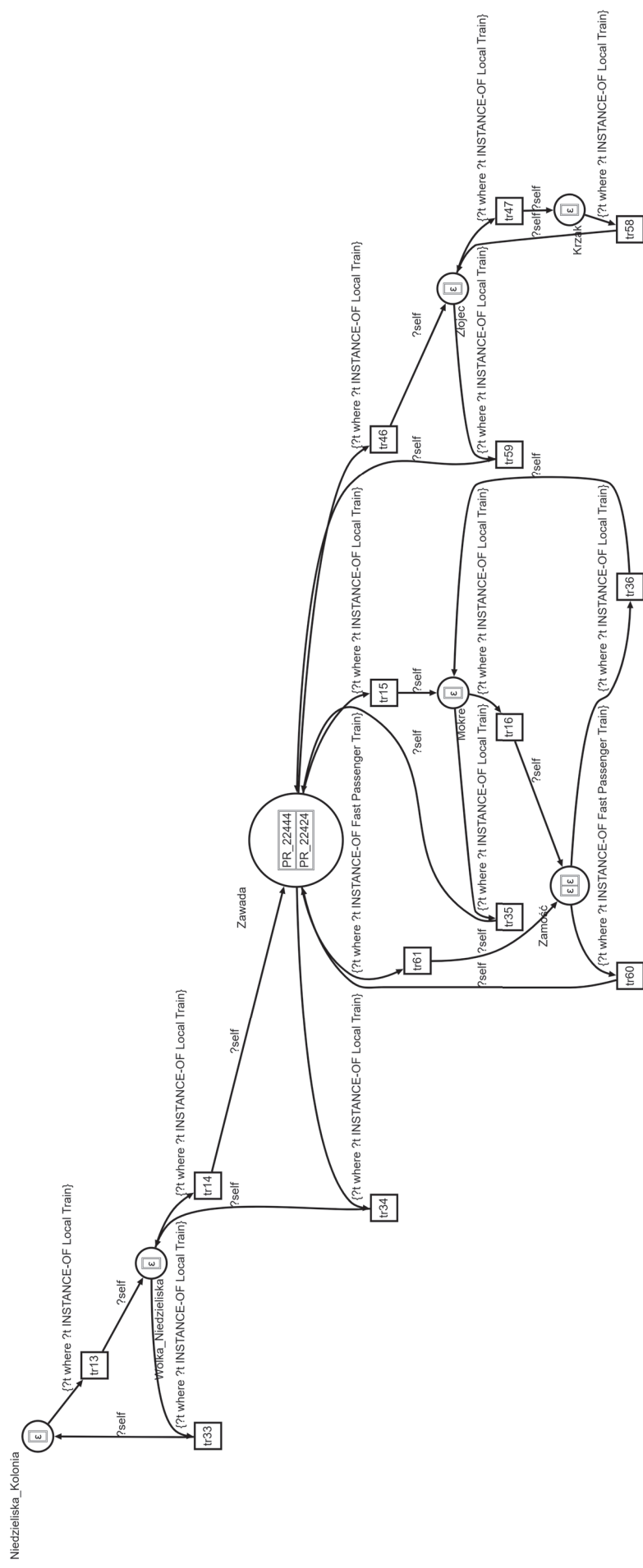
- symbolic artificial intelligence—a set of artificial intelligence methods on high-level (symbolic) representations of problems (Thomason 2003)
- explainable artificial intelligence—an approach to creating artificial intelligence algorithms that enable understanding/interpretation of the path to the final result (Longo et al. 2024)
- computing with words—a computational system in which the subject of calculations are mainly words, expressions and sentences taken from natural language (Zadeh 2012)

In the next step of the research, we are planning to take into account time dependencies in the proposed Petri net model. In case of transport procedures, we will be able to identify possible states of procedure blockades or bottlenecks. There have been considered time-dependent Petri net models in the literature (Popova-Zeugmann 2013). Moreover, we are developing PyPNOG (Python Petri Nets over Ontological Graphs)—the Python toolkit for dealing with Petri nets over ontological graphs. This toolkit will enable us to implement analysis, simulation and validation tools for Petri Nets over Ontological Graphs.



**Figure 6.** The fragment of the multi-instancely marked Petri net over ontological graphs modeling of passenger rail transport in the Zamość region (an initial marking)





**Figure 7.** The fragment of the multi-instancely marked Petri net over ontological graphs modeling of passenger rail transport in the Zamość region (after firing transitions *tr59* and *tr35*)

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