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Application of MS-GARCH Class Models in the Analysis of EUR/PLN Exchange Rate Risk in the Context of Poland's Monetary Integration and the ERM II Mechanism

Patryk Kołbyko
Maria Curie-Skłodowska University in Lublin, Poland

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Abstract

The aim of this article is to identify a hypothetical central EUR/PLN exchange rate within the ERM II framework and to assess the risk of exceeding the permissible fluctuation band for the 2025–2028 horizon. The study uses daily exchange rate data from 2020–2024 and applies nonlinear models of the MS-GARCH (Markov-Switching GARCH) class in both homogeneous and heterogeneous variants, estimated using the quasi-maximum likelihood method. The research procedure included the selection of optimal P and Q parameters according to the AIC information criterion, the identification of periods characterized by heightened volatility regimes, and the forecasting of exchange rate volatility using the MS-GARCH model combined with Monte Carlo simulations for logarithmic returns. The results indicate an optimal ERM II central rate of PLN 4.53, with the heterogeneous MS-GARCH model revealing 21 days of elevated risk of exceeding the upper threshold in November and December 2027. The study contributes an empirical, atheoretical approach to determining the ERM II central rate, filling a gap in the analysis of emerging economies outside the euro area.

Keywords: euro area, ERM II, monetary integration process, EUR/PLN exchange rate risk

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Introduction

The Economic and Monetary Union (EMU) constitutes one of the fundamental elements of the European Union's structure and an essential condition for the full institutional integration of a member state with the other EU countries. The EMU was established under the Maastricht Treaty — the Treaty on European Union of 1992 — and its implementation of the institutional integration of the member states, it was based on a strategy of gradualism, the origin of which can be found in the Delors Report (Bijak-Kaszuba 2012). This strategy is based on three stages, with Stage III being implemented since January 1, 1999, and concerning monetary integration through, among other things, the introduction of the euro, the loss of full monetary sovereignty by the member state, and a common policy within the framework of the European Central Bank and the European System of Central Banks. Poland, as one of six (excluding Denmark) member states, maintains a derogation status, and the decision to join the ERM II mechanism as an element of a conditionally determined accession strategy is normative in nature and depends on political decisions and social approval. At present, the majority of Polish society is opposed to monetary integration, as indicated by survey data. Since Poland's accession to the structure of the European Union on May 1, 2004, under the Treaty of Athens (signed on April 16, 2003), and its acceptance of the *Acquis communautaire*,

E-mail addresses and ORCID digital identifiers of the authors
Patryk Kołbyko • e-mail: patryk.kolbyko2000@gmail.com • ORCID: 0000-0001-6550-8452

successive governments, despite fulfilling the remaining nominal convergence criteria (in accordance with the Treaty on the Functioning of the European Union): price stability, public finance stability, and a long-term interest rate below the reference threshold, have neither decided nor declared an intention to join ERM II. Officially, the Convergence Report of 2016 confirmed the fulfillment of the remaining criteria; in 2018 the criterion of the long-term interest rate was not met, while in 2020 the criterion of price stability was not fulfilled. According to the Convergence Reports of 2022 and 2024, Poland does not meet any of the criteria.[1] Historically, there were cases such as Italy and Belgium (in the year 1999), which did not meet the public finance stability criterion (the case of Greece in 2001 is similar, but it formally met the criterion after data corrections by Eurostat), and despite this, the Council of the European Union decided to recognize the criteria as met through interpretation. Therefore, this study focuses exclusively on the conditional forecast (without the mechanism of intramarginal interventions) of the risk of nominal EUR/PLN exchange rate instability in the years 2025–2028 and on the identification of the hypothetical level of the ERM II central exchange rate, which would be conditioned by the periods of the EUR/PLN rate being in a low-volatility regime.

The aim of this study is to identify the ERM II central exchange rate with a symmetric range of permissible deviations of the nominal EUR/PLN rate and to analyze the risk of exceeding the ERM II corridor through volatility forecasting using MS-GARCH models for the period 2025–2028.

To achieve the research objective, data on the nominal EUR/PLN exchange rate process and first-order logarithmic differences (Close-to-Close) in a daily sequence (excluding holidays and weekends) will be used. The data were obtained from the Yahoo Finance database, access to which was provided by the "quantmod"[2] library in RStudio. This study is based on the following procedure:

- preparation of descriptive statistics: measures of asymmetry and dispersion (kurtosis) for the standardized residuals of the base regression model
- examination of the conformity with the probability distribution and visualization of the distribution
- identification of the variance clustering phenomenon
- identification of optimal parameters P and Q for GARCH(P,Q)-std using a selective-iterative method with respect to the AIC information criterion as an indicator of predictive ability
- selection of two GARCH-class models in terms of their fit to the data for constructing the variants: a homogeneous MS-GARCH model with a common variance family and a heterogeneous MS-GARCH model with different variance families
- identification of observations in the low-volatility regime, taking into account the regimes of increased volatility under both variants of the MS-GARCH model (in terms of the probability of transition to a higher-variance regime and exceeding the 80th percentile volatility threshold), and then determining the central parity (the mean from the periods of low conditional volatility)
- simulation of the log return path using the Monte Carlo algorithm and the MS-GARCH model and forecasting volatility for the 5%–95% percentile range for the years 2025–2028, which made it possible to identify the risk of exceeding the permissible deviation range from the ERM II central parity under both variants of the MS-GARCH model.[3]

Research on the measurement of the ERM II central parity and the equilibrium exchange rate in the academic literature is characterized by strong theoretical coherence, which applies, among other models, to the measurement of the Natural Exchange Rate (NATREX) or the Fundamental Equilibrium Exchange Rate (FEER), as well as to complex dynamic models with a cointegration process used for measuring the Behavioral Equilibrium Exchange Rate (BEER) or the Permanent Equilibrium Exchange Rate (PEER). This study aims to fill the research gap concerning an atheo-

1. See: Convergence Reports by European Commision Directorate-General for Economic and Financial Affairs, available at https://economy-finance.ec.europa.eu/euro/enlargement-euro-area/convergence-reports_en#convergence-reports.

2. Jeffrey A. Ryan et al. (2025). quantmod. More info at: https://cran.r-project.org/web/packages/quantmod/index.html.

3. The calculations were performed using the package rugarch — Galanos A (2025). rugarch: Univariate GARCH models. R package version 1.5-4. More info at: https://cran.r-project.org/web/packages/rugarch/index.html.

retical and strongly empirically coherent approach to the measurement of the ERM II central parity for the nominal exchange rate, which is the rate determined in institutional practice. To address this research gap, particularly from the perspective of the stability of the EUR/PLN exchange rate valuation under a floating exchange rate regime, the leading analytical tool will be the nonlinear MS-GARCH model.

In the literature, the issue of modeling exchange rate risk in the context of the ERM II mechanism has so far focused mainly on *ex post* analysis for economies already participating in the exchange rate system, using classical GARCH models or their simple modifications (Hansen and Lunde 2005; Marisetty 2024). The research gap, however, concerns the lack of empirical studies employing MS-GARCH-class models for the identification of volatility regimes and the forecasting of the risk of exceeding the permissible fluctuation band for emerging economies remaining outside ERM II, including Poland. This study represents an attempt to fill this gap by applying MS-GARCH models to forecast the volatility of the nominal EUR/PLN exchange rate and to determine the ERM II central rate for the 2025–2028 horizon.

Of the 27 countries forming the European Union, seven — Bulgaria, the Czech Republic, Denmark, Hungary, Poland, Romania, and Sweden — are not members of the Monetary Union. With the exception of Denmark, which, under the opt-out clause (concerning monetary integration) established in the Edinburgh Agreement of 1992, is partially exempt from the implementation of Stage III of the EMU, the remaining six countries, representing both developed (Sweden) and emerging economies, formally maintain a derogation status. The fulfillment of the monetary convergence criterion, which is conditional upon entry into ERM II, depends on a political decision and therefore has a strongly normative character. Nevertheless, in 2022 Croatia joined the Euro Area, and Bulgaria has declared its intention to enter the Monetary Union by 2027, which indicates that the political demand and willingness for further institutional integration within the European Union remain alive. Therefore, for researchers and economic practitioners dealing with emerging economies outside the Euro Area, it is important to determine the key element of monetary convergence, the ERM II central exchange rate along with the symmetric 15% fluctuation band. This study provides an analytical tool, using MS-GARCH class models and the EUR/PLN exchange rate process from 2020 to 2024 as an example, to identify periods of the low-volatility regime, which in turn make it possible to extract the average level of the nominal exchange rate, the central rate, and to develop a volatility forecast for the period 2025–2028.

The EU Convergence Report, prepared by the European Council, is published every two years; therefore, the results of this study on the EUR/PLN exchange rate risk forecast and the application of this procedure in further research on other emerging economies will allow results to be compared with the EU Convergence Reports to be published in 2026 and 2028, which will cover economies joining the ERM II mechanism during that time. At present, the application of MS-GARCH-class models for conditional volatility forecasting for a fixed central rate with an allowable fluctuation band is particularly relevant for Bulgaria, which participates in ERM II and whose central rate was established in 1997 under the monetary strategy of a currency board at the level of 1.95583 EUR/BGN. The procedure for identifying the optimal central rate in this study also constitutes a methodological proposal for economic practitioners and researchers from countries that officially declare their intention to join ERM II.

1 Literature review

In the research literature, there is a clear research gap concerning the measurement of the optimal ERM II central exchange rate for emerging economies that are part of the European Union but remain outside the Monetary Union, as well as the analysis of volatility within the permissible fluctuation band in ERM II using atheoretical nonlinear models such as MS-GARCH. As emphasized by the Polish Economic Institute, the analysis and forecasting of exchange rates constitute a difficult challenge and are primarily focused on the issue of the equilibrium exchange rate.[4]

4. See: Miesięcznik Makroekonomiczny 1/2024, Polski Instytut Ekonomiczny, February 2, 2024, available at https://pie.net.pl/miesiecznik-makroekonomiczny-1-2024/.

Researchers (Melecky and Komarek 2007; Pastucha 2024) as well as state institutions and international organizations (Belfrage, Hansson, and Vredin 2023; Filardo, Gelos, and McGregor 2022) use for this purpose complex models with the imputation of macroeconomic variables (most often selected based on heuristic knowledge of macroeconomic processes), complex models identifying the cointegration process, or models characterized by strong theoretical coherence through structural equations shaping the long-term equilibrium of the exchange rate, most often measured multilaterally within a currency basket. Similarly, the European Central Bank suggests setting the central rate at the level of the real equilibrium exchange rate.[5] The approach of researchers and institutions allows for the identification of, among others, the Behavioral Equilibrium Exchange Rate and the Fundamental Equilibrium Exchange Rate. A particularly interesting case was the use of the NATREX (Natural Exchange Rate) model, which meets the assumptions of Thirlwall's Law (Oreiro 2023) — external balance (balance of payments equilibrium) and internal balance (general equilibrium) — as a guideline for determining the ERM II central rate by the Central Bank of Latvia (Anghel et al. 2014).

This study emphasizes the role of parsimony and efficiency in constructing forecasting models for exchange rate volatility risk, pointing to the application of GARCH-class models. The advantage of parsimony in the imputation of theory into risk forecasting models was noted by Meese and Rogoff (1983), which remains relevant today, considering more recent replicated studies (de Mendonca, Vereda, and Araujo 2025; Kilic 2025). However, at present, the popularity of GARCH-class models and their extension to include the identification of regime transition probabilities (Markov Chain) in MS-GARCH models in the context of risk analysis and the ERM II mechanism is almost negligible, and their use can be found mainly in older studies (Fidrmuc and Horváth 2008; Frömmel 2006).

2 Research method

The starting point for the analysis of EUR/PLN exchange rate risk is the analysis of the process of logarithmic returns for data in a daily sequence from the beginning of 2020 to the end of 2024. For the daily (nominal) spot exchange rate EUR/PLN, $\{P_t\}_{t=1}^{T}$, the logarithmic returns are defined as

$$r_t = \log(P_t) - \log(P_{t-1}), \qquad t = 2, \ldots, T. \tag{1}$$

First, a regression equation was constructed consisting of the following components: the conditional mean, μ_t, and the standardized regression residuals, ε_t, with the product of the scaling component representing the standardized shock, σ_t, and the conditional standard deviation:

$$r_t = \mu_t + \varepsilon_t, \tag{2}$$

$$\varepsilon_t = \sigma_t z_t, \qquad z_t \sim i.i.d.\ F(0,1;\psi), \tag{3}$$

where F denotes a standardized density distribution with ψ degrees of freedom. For such a defined process of standardized residuals, descriptive statistics were estimated: measures of asymmetry (third-order central moment), namely skewness, and measures of dispersion (fourth-order central moment), namely kurtosis. Subsequently, the Jarque-Bera test was conducted to identify the distribution of standardized residuals used in the later stages of estimating GARCH(P,Q)-class models. The squares of standardized residuals were used to examine conditional volatility over time and the occurrence of autocorrelation processes associated with the phenomenon of variance clustering (the variance grouping effect).

To identify the occurrence of variance clustering over time, the ARCH-LM test (Engle's test) and the Ljung-Box portmanteau test were applied. Given the study's foundation on frequentist inference, for an arbitrarily set Type I error risk threshold (α), the statistics of both tests falling within the critical region confirm the presence of variance clustering over time. Tests for variance clustering provide important information about volatility; however, if volatility is significant across

5. See: Policy Position Of The Governing Council Of The European Central Bank On Exchange Rate Issues Relating To The Acceding Countries. European Central Bank, December 18, 2003, document available at https://www.ecb.europa.eu/pub/pdf/other/policyaccexchangerateen.pdf.

successive lags — the persistence process — they are not able to indicate the optimal parameter of conditional variance for the ARCH(P) model. Furthermore, the Augmented Dickey-Fuller (ADF) test for the presence of a unit root was conducted to examine the stationarity of the logarithmic returns process.

The identification of the optimal conditional variance parameters P and Q was based on the explanatory power of the model, the Akaike Information Criterion (AIC), which is recommended (Chakrabarti and Ghosh 2011) for forecasting and univariate models, rather than the Bayesian Information Criterion (BIC), which is more robust to complex model structures. To identify the optimal structure of the GARCH(P,Q) model, a selective-iterative procedure was applied, as used, among others, by Kołbyko (2024) (who employed it for identifying the optimal SARIMA/ARIMA model). This approach constitutes an alternative to the classical Box-Jenkins method (in which the first stage, identification, is based on ACF and PACF tests) and involves the iterative estimation of models and the selection of the model with the lowest value of the adopted information criterion.

The canonical GARCH(P,Q) model proposed by Bollerslev (1986), applied in this study and in its further extended specifications, takes the form consisting of a lagged volatility process — the conditional variance of the ARCH(P) model — and the GARCH(P,Q) component, i.e., a lagged process of its own volatility up to order Q. The structure of the GARCH(P,Q) model is as follows:

$$\sigma_t^2 = \omega + \underbrace{\sum_{p=1}^{P} \alpha_p \varepsilon_{t-p}^2}_{\text{ARCH}} + \underbrace{\sum_{q=1}^{Q} \beta_q \sigma_{t-q}^2}_{\text{GARCH}}. \tag{4}$$

For the classical form of the GARCH(P,Q) model, specific assumptions are made regarding the parameter of the long-term mean — the "unconditional variance," $\omega > 0$, as well as the coefficients of the lagged conditional variance process: α_p, β_q, ≥ 0, and the sum of the coefficients for subsequent lags of both components of the conditional variance is less than 1:

$$\sum_{p=1}^{P} \alpha_p + \sum_{q=1}^{Q} \beta_q < 1. \tag{5}$$

The fulfillment of these assumptions makes it possible to achieve conditional stationarity (a model misspecification resulting in $\alpha + \beta = 1$ would prevent this) of the GARCH(P,Q) model, as well as the predictive capability for risk based on conditional volatility ($\omega = 0$ would also prevent this). Therefore, the elementary condition of an optimal GARCH(P,Q) model is the adoption of the above preliminary assumptions to obtain a positive unconditional long-term variance:

$$\hat{\sigma}_t^2 = \frac{\omega}{1 - \left(\sum \alpha_p + \sum \beta_q\right)}. \tag{6}$$

If the cumulative coefficients for the lagged processes of conditional variance take the value of 1, then the Integrated GARCH (IGARCH) model should be applied. For estimation, the quasi-maximum likelihood (QMLE) method was used, which employs as its objective function the maximization of the log-likelihood for the assumed Student's t-distribution of the conditional residual component. Let θ denote the set of unknown parameters in the GARCH(P,Q) model; then, the conditional density distribution of logarithmic returns given past information, $\mathcal{F}_{t-1}$, is expressed as

$$f(r_t|\mathcal{F}_{t-1}; \theta, \psi) = \frac{1}{\sigma_t(\theta)} f_z\left(\frac{r_t - \mu}{\sigma_t(\theta)}; \psi\right). \tag{7}$$

The standardized density function of the Student's t-distributed residuals, f_z, is determined from

$$f_z(u, \nu) = \frac{\Gamma\left(\frac{\upsilon+1}{2}\right)}{\sqrt{\pi(\upsilon-2)}\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{u^2}{\upsilon - 2}\right)^{-\frac{\upsilon+1}{2}}. \tag{8}$$

Where the likelihood function is

$$\mathcal{L}_T(\theta,\psi|r) = \prod_{t=1}^{T} f(r_t|\mathcal{F}_{t-1};\theta,\psi). \tag{9}$$

Consequently, the conditional log-likelihood is

$$\ell_T(\theta,\psi) = \log \mathcal{L}_T(\theta,\psi|r) = -\sum_{t=1}^{T}\left[\log\sigma_t(\theta) + \rho_\psi\left(\frac{r_t-\mu}{\sigma_t(\theta)}\right)\right] + C(\psi). \tag{10}$$

Where the function $\rho_\psi = -\log f_z(u,\nu)$, and the constant $C(\psi)$, which is identical for all observations and does not affect the estimation of the set of unknown parameters, serves only to normalize the standardized distribution (depending solely on ψ) and does not influence the maximization of the log-likelihood. For the QMLE estimator, the maximization of the log-likelihood function is achieved through the selection of parameters that yield the maximum value

$$(\hat{\theta},\hat{\psi}) \in \arg\max_{\theta,\psi} \ell_T(\theta,\psi). \tag{11}$$

The procedure for identifying the optimal GARCH(P,Q) model based on the selective-iterative method employs a grid search structure as the candidate space

$$G = \{(P,Q)\colon P = 0,\ldots,P_{\max}, Q = 0,\ldots,Q_{\max}\}\backslash\{(0,0)\}. \tag{12}$$

With the arbitrary setting of $P_{\max}, Q_{\max} = 5$. The identification of the model's optimal parameters was based on the AIC criterion in accordance with the initial assumptions, where the value of the explanatory indicator is determined as $\mathrm{AIC} = -2\hat{\ell} + 2k$, while the sequential selection using the grid search selects those model parameters for which the GARCH(P,Q) achieves the lowest value of the AIC information criterion, such that

$$(P^*,Q^*) \in \arg\min_{(P,Q)\in G} \widehat{\mathrm{AIC}}(P,Q). \tag{13}$$

The study was extended to include extended specifications of the GARCH(P,Q) model without internal selection but using the already obtained parameters P and Q. The applied models were: GJR-GARCH, eGARCH, APARCH, and TGARCH. For all specifications, the following were prepared:

- the conditional Value-at-Risk (stochastic VaR for $\alpha = 5\%$), which was determined (for the lower and upper quantiles: quantile $\in$ {left, right}) as:

$$\mathrm{VaR}^{\text{quantile}}_{t\text{-Student},\alpha} = \hat{\mu}_t + \hat{\sigma}_t q^{\text{quantile}}_{z,\alpha}(\hat{\psi}) \tag{14}$$

- the News Impact Curve (NIC) function:

$$g(z_{t-1}) \coloneqq E[\sigma_t^2]\varepsilon_{t-1} \tag{15}$$

- the Expected Shortfall (ES) for the left tail of the distribution:

$$\mathrm{ES}_{t\text{-Student},\alpha} = E[r_t|r_t \le \mathrm{VaR}_{t\text{-Student},\alpha}] = \mu_t + \sigma_t \cdot E[z_t|z_t \le q_{z,\alpha}] \tag{16}$$

- where for $c_\alpha(f_z,\psi) \equiv E[z_t|z_t \le q_{z,\alpha}]$, the Student's t-distribution was adopted, along with the number N of Monte Carlo samples:

$$\hat{c}_\alpha = \frac{1}{N_\alpha}\sum\nolimits_{n\colon z^{(n)} \le q_{z,\alpha}} z^{(n)}, \qquad z^{(n)} \sim f_z(\cdot;\hat{\psi}) \tag{17}$$

The selection of m-specifications of the GARCH-class model for constructing the MS-GARCH model with homogeneous regimes (the same m-specification of variance with different parameters) and with heterogeneous regimes (different m-specifications of variance) was based on the lowest values of the AIC information criterion. Two models with the best predictive ability were identified, along with the construction of switching between two, $K = 2$, regimes in: the homogeneous MS-GARCH model, where $\nu\left(\cdot,\vartheta^{(k)}\right)$ and $\vartheta^{(1)} \neq \vartheta^{(2)}$, and in the heterogeneous MS-GARCH model, where the variance families in the regimes differ: $\nu_1 \neq \nu_2$. Consequently, the variance-updating functions for the given regimes in the MS-GARCH models are defined as:

$$\sigma_t^{2,(k)} = g\left(\varepsilon_{t-1}, \sigma_{t-1}^{2,(k)}; \vartheta^{(k)}\right) \qquad g_1 \equiv g_2 \quad \text{(homogeneous MS-GARCH)}, \tag{18}$$

$$\sigma_t^{2,(k)} = g_k\left(\varepsilon_{t-1}, \sigma_{t-1}^{2,(k)}; \vartheta^{(k)}\right) \qquad g_1 \not\equiv g_2 \quad \text{(heterogeneous MS-GARCH)}. \tag{19}$$

The hidden regime-switching process (Markov chain), $S_t \in \{1, 2\}$, is based on the conditional transition probability

$$\mathcal{P}(S_t = j \mid S_{t-1} = 1) = p_{ij}, \tag{20}$$

where the transition probability matrix between the regimes of the Markov chain is

$$P = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}, \qquad \text{where } p_{i1} + p_{i2} = 1. \tag{21}$$

The stationary distribution, π, satisfies $\pi^{\mathrm{T}}P$. Then, the conditional model of the log return process within the Markov regime-switching structure is defined as: $r_t = \mu + \sigma_t^{(k)} + z_t^{(k)}$. The conditional probability density function of the log returns in the MS-GARCH models takes the form

$$f(r_t|\mathcal{F}_{t-1}) = \sum_{k=1}^{2} \underbrace{\mathbb{P}(S_t = k|\mathcal{F}_{t-1})}_{\pi_{t|t-1}(k)=\sum_{k=1}^{2}\alpha_{t-1}(k)p_{ik}} \cdot f_k\left(r_t|\mathcal{F}_{t-1}; \vartheta^{(k)}, \psi^{(k)}\right) \tag{22}$$

where the likelihood function is

$$\mathcal{L}_T\left(\{p_{ij}\}, \{\vartheta^{(k)}\}, \{\psi^{(k)}\}\right) = \prod_{t=1}^{T} f(r_t|\mathcal{F}_{t-1}) \tag{23}$$

Consequently, the log-likelihood function is

$$\ell_T = \sum_{t=1}^{T} \log\left[\sum_{k=1}^{2} \pi_{t|t-1}(k) \frac{1}{\sigma_t^{(k)}\left(\vartheta^{(k)}\right)} f_z\left(\frac{r_t - \mu}{\sigma_t^{(k)}\left(\vartheta^{(k)}\right)}; \psi^{(k)}\right)\right] \tag{24}$$

The final stage involved identifying periods of high EUR/PLN exchange rate volatility within the analyzed period by setting a threshold, the 80-percentile of the conditional standard deviation of the log return process, under both MS-GARCH models. This made it possible to isolate periods of relatively low volatility and to determine, within this set, the proposed central exchange rate with a symmetric 15% fluctuation band. The established parity was determined from M-observations as the 60-percentile of observations representing the stable EUR/PLN exchange rate level (stable EUR/PLN rate: $\mathrm{ExR}_{(j)}^{\mathrm{stable}}$):

$$\text{parity} = \frac{1}{M} \sum_{m=1}^{M} \mathrm{ExR}_{(j)}^{\mathrm{stable}}. \tag{25}$$

Consequently, the determined ERM II fluctuation band is

$$\mathrm{ERM}^{\mathrm{lower}} = 0.85 \cdot \text{parity} \quad \text{and} \quad \mathrm{ERM}^{\mathrm{upper}} = 1.15 \cdot \text{parity}. \tag{26}$$

The central path, the point forecast of the EUR/PLN exchange rate under both MS-GARCH models, was determined as

$$\widehat{\mathrm{ExR}}_{T+h}^{(n)} = \mathrm{ExR}_T \cdot e^{\left(\sum_{j=1}^{J} r_{T+j}^{(n)}\right)}. \tag{27}$$

For the path of future log returns from the fitted MS-GARCH models: $\left\{r_{T+H}^{(n)}\right\}_{h=1}^{H}$ (where the number of Monte Carlo simulation paths $n = 1, ..., N$). Subsequently, the 5% deviation interval of the point forecast of the EUR/PLN exchange rate was determined as the range between the 5% and 95% quantiles from the simulations. The point forecast itself was then rescaled to the initial condition

$$\mathrm{ExR}_{T+h}^{(n)} = \frac{\text{parity}}{\mathrm{ExR}_T} \widehat{\mathrm{ExR}}_{T+h}^{(n)}. \tag{28}$$

3 Research results

The secondary data for the time series of the nominal EUR/PLN exchange rate in a daily sequence for the period from the beginning of 2020 to the end of 2024 were obtained using the "quantmod" library from the CRAN repository in RStudio, sourced from the Yahoo Finance database.

This study is based on frequentist inference; therefore, hypothesis testing relies on distribution-based tests, with an arbitrarily set Type I error risk threshold of $\alpha = 0{,}05$. A test statistic falling within the critical region for the chosen significance level is marked with an asterisk (*).

After conducting the goodness-of-fit test for the Gaussian probability distribution, the result KS ≈ 95.99 indicated a non-normal distribution for the process of log first differences of the EUR/PLN exchange rate. Descriptive statistics of the third- and fourth-order central moments indicated the presence of leptokurtosis and moderate skewness, with Skewness ≈ 0.28 and Kurtosis ≈ 3.81. Therefore, the distribution applied in the GARCH and MS-GARCH class models is the symmetric Student's t-distribution ("std"). Figure 5 in Appendix visualizes the distribution results of log returns in four panels: Boxplot, Quantile-Quantile plot, Histogram (frequency series determined using the Freedman-Diaconis rule) with the theoretical Gaussian distribution and the empirical distribution — Epanechnikov kernel density estimator — as well as the cumulative distribution plot comparing the theoretical normal CDF with the empirical CDF. The visualized results clearly highlight the phenomenon of leptokurtosis in the log return process.

According to the adopted methodological procedure for identifying the parameters of the GARCH(P,Q)-std model based on the selective-iterative method with grid search for the lowest AIC information criterion value, the results indicated the optimal GARCH(1,1)-std model. A comparison of the AIC criterion results with other parameter combinations can be observed in table 1. The selection of the GARCH(1,1) model and its high predictive capability are consistent with the findings of Hansen and Lunde (2005), Marisetty (2024), and Makore and Chikutuma (2025).

Table 1. Results of the AIC information criterion values for individual parameter combinations of the standard GARCH(P,Q)-std model

	$Q = 0$	$Q = 1$	$Q = 2$	$Q = 3$	$Q = 4$	$Q = 5$
$P = 0$	NA	−10,772.150	−10,770.228	−10,768.234	−10,770.136	−10,768.149
$P = 1$	−10,821.411	**−10,878.636**	−10,876.647	−10,875.446	−10,873.422	−10,871.269
$P = 2$	−10,853.569	−10,876.586	−10,874.647	−10,874.191	−10,872.814	−10,870.711
$P = 3$	−10,857.982	−10874.553	−10,872.615	−10,873.222	−10,871.106	−10,869.032
$P = 4$	−10,862.260	−10,872.678	−10,870.702	−10,871.106	−10,869.106	−10,867.032
$P = 5$	−10,867.172	−10,871.434	−10,869.577	−10,869.229	−10,867.229	−10,865.234

Note: The combination of parameters P and Q with the best fit of the GARCH model to the data is highlighted in bold.

Figure 6 visualizes the results of the conditional quantiles: 5% (lower risk tail) and 95% (upper risk tail). The results of the GARCH models indicate exceedances of both the 5% lower and upper risk tails in response to structural changes and global imbalances, including the COVID-19 pandemic, the Russian Federation's invasion of Ukraine, and the parliamentary elections. Models with an asymmetric response of conditional volatility to the sign of the shock, GJR-GARCH and EGARCH, exhibit a wider VaR bandwidth (the range between the risk tails), resulting particularly from the identification of the leverage effect, i.e., the impact of appreciation shocks (negative shocks under direct spot rate quotation) on volatility. The most conservative model turned out to be the TGARCH model, with 49 exceedances (approximately 3.75%), while closer to the 5% level were the GARCH and GJR-GARCH models, each with 53 exceedances (approximately 4.06%). The study was extended to include verification of the risk coverage ability of the models using the following tests: Kupiec's POF (Unconditional Coverage), Christoffersen's (Conditional Coverage), and the Engle-Manganelli (Dynamic Quantile) tests. According to the Kupiec test, only for the TGARCH model was the null hypothesis of violation frequency, relative to the assumed α level, rejected. Furthermore, the results of the CC and DQ tests indicated no clustering of exceedances, implying that the models are correctly calibrated, with the best-performing ones being GARCH, EGARCH, and GJR-GARCH. Figure 8 visualizes the Expected Shortfall, the average loss magnitude after exceeding the 5% VaR, across the grid of analyzed GARCH models.

Figure 7 visualizes the News Impact Curves (NIC) for the examined GARCH models, which illustrate the effect of innovations from period $t-1$ on conditional volatility. Although this allows for the identification of the leverage effect and the impact of shock sign asymmetry, the results did not indicate any significant asymmetry, even in the EGARCH, GJR-GARCH, and TGARCH models. However, the EGARCH and TGARCH models display a sharper response of conditional variance, as reflected in the steeper shape of the curves.

Figure 9 visualizes the results of the conditional variance forecast (left panel) and the unconditional standard deviation (right panel) up to 2027. The lowest value of unconditional standard deviation was shown by the EGARCH(1,1)-std model (after transformation from log-space), amounting to approximately 0.382%, whereas the highest long-term standard deviation was produced by the APARCH(1,1)-std model, approximately 0.414%.

For the grid of GARCH-class models with parameters $P = 1$ and $Q = 1$ and a conditional Student's t-distribution, the results of the AIC information criterion were prepared (table 2). The results indicated that the optimal model is the asymmetric GJR-GARCH(1,1)-std, while the suboptimal models are the APARCH(1,1)-std and the classical GARCH(1,1)-std. Accordingly, for the construction of the homogeneous MS-GARCH model, the regimes for a single variance family were conditioned by the GJR-GARCH(1,1) model, whereas in the heterogeneous model, the regimes for different variance families were conditioned by the GJR-GARCH(1,1) model (regime 1) and the classical GARCH(1,1) model (regime 2).

Table 2. Summary of the estimated GARCH-class models and the results of the AIC information criterion

Model	AIC
GJR-GARCH(1,1)-std	**−10,880.49**
APARCH(1,1)-std	−10,879.04
GARCH(1,1)-std	−10,878.64
TGARCH(1,1)-std	−10,872.40
EGARCH(1,1)-std	−10,871.07

Note: AIC for best model is highlighted in bold.

After constructing and estimating the models, the results showed that the heterogeneous model demonstrated a better fit to the data in terms of explanatory power — as measured by the AIC criterion (homogeneous: AIC = −10,858.92, heterogeneous: AIC = −10,860.92). In both models, the first regime corresponds to higher conditional volatility, while regime 2 represents lower conditional volatility. However, the results for the average conditional standard deviation show that in the homogeneous model, both regimes exhibit lower volatility compared to the conditional standard deviation results in both regimes of the heterogeneous model. For the homogeneous model: $\text{regime}_{1,\text{homogeneous}} \approx 0.38\%$ and $\text{regime}_{2,\text{homogeneous}} \approx 0.31\%$, whereas for the heterogeneous model: $\text{regime}_{1,\text{heterogeneous}} \approx 0.60\%$ and $\text{regime}_{2,\text{heterogeneous}} \approx 0.56\%$.

Figure 1 (on next page) consists of two panels: the first panel (upper) visualizes the filtered process of the conditional transition probability between regimes, where a result closer to 1 indicates a switch from the low-volatility regime to the higher-volatility regime. The results of the homogeneous and heterogeneous MS-GARCH(1,1)-std models differ only slightly, as shown in the lower panel. However, the lower panel indicates that during key periods — the onset of the COVID-19 pandemic, the beginning of the Russian Federation's invasion of Ukraine, and the results of the parliamentary elections in Poland in 2023 — the heterogeneous MS-GARCH model more accurately captured the risk of increased volatility in the log returns of the nominal EUR/PLN exchange rate.

Due to the similar results of the filtered conditional volatility forecast between the heterogeneous and homogeneous MS-GARCH models, figure 2 (on next page) visualizes the process of differences between the results over the analyzed period.

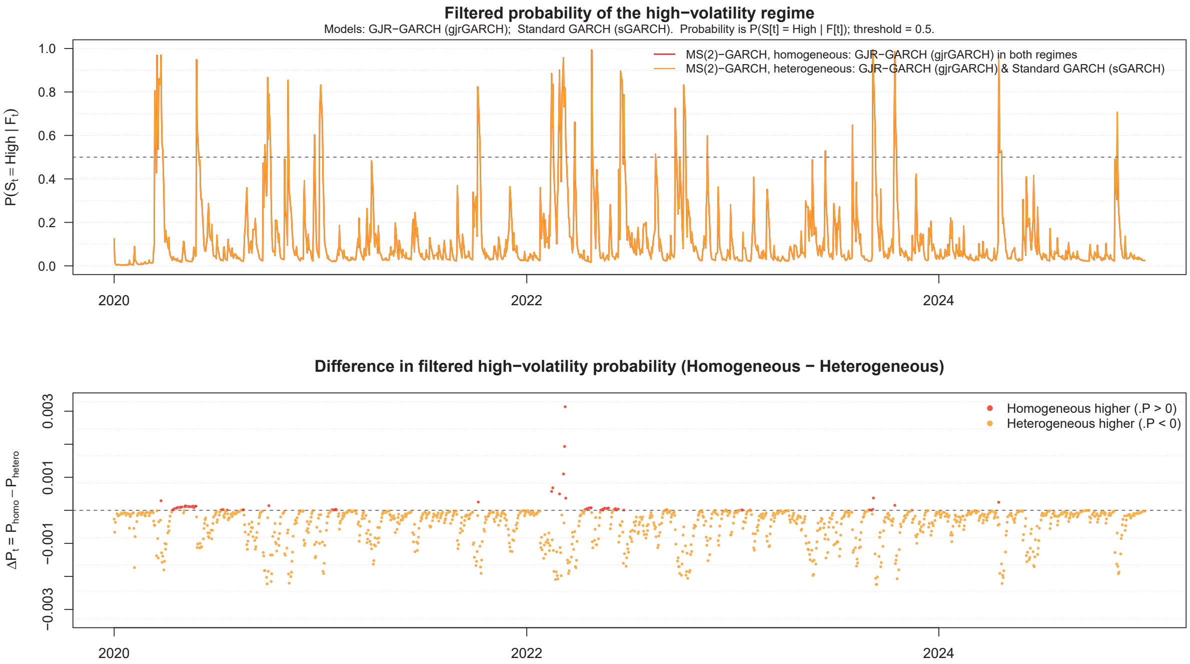

Figure 1. Filtered process of the transition probability to the higher-volatility regime under both variants of the MS-GARCH model

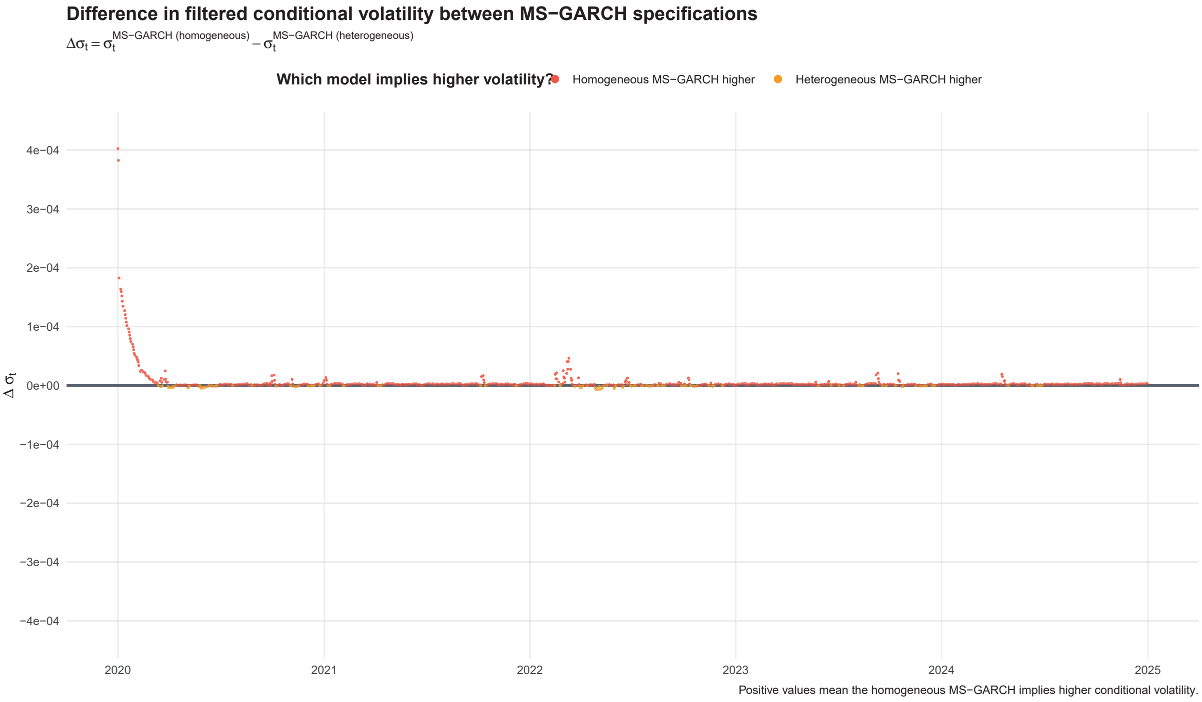

Figure 2. Differences in conditional standard deviations between the homogeneous and heterogeneous MS-GARCH models

According to the methodological procedure, for the developed MS-GARCH models, periods of higher volatility were identified (figure 3) when, for a given period, the conditional probability of transition to the higher-volatility regime was less than or equal to 0.5, and in the higher-volatility regime, the conditional standard deviation was above the 80th percentile of the distribution under both variants of the MS-GARCH model (periods of elevated volatility could overlap).

For the 60th percentile of the EUR/PLN exchange rate distribution during periods of stable volatility (taking into account the periods of elevated volatility in both variants of the MS-GARCH models), the average level was determined, which constitutes one of the two main elements of the analysis in this study, the ERM II central exchange rate with a symmetric ±15% fluctuation band.

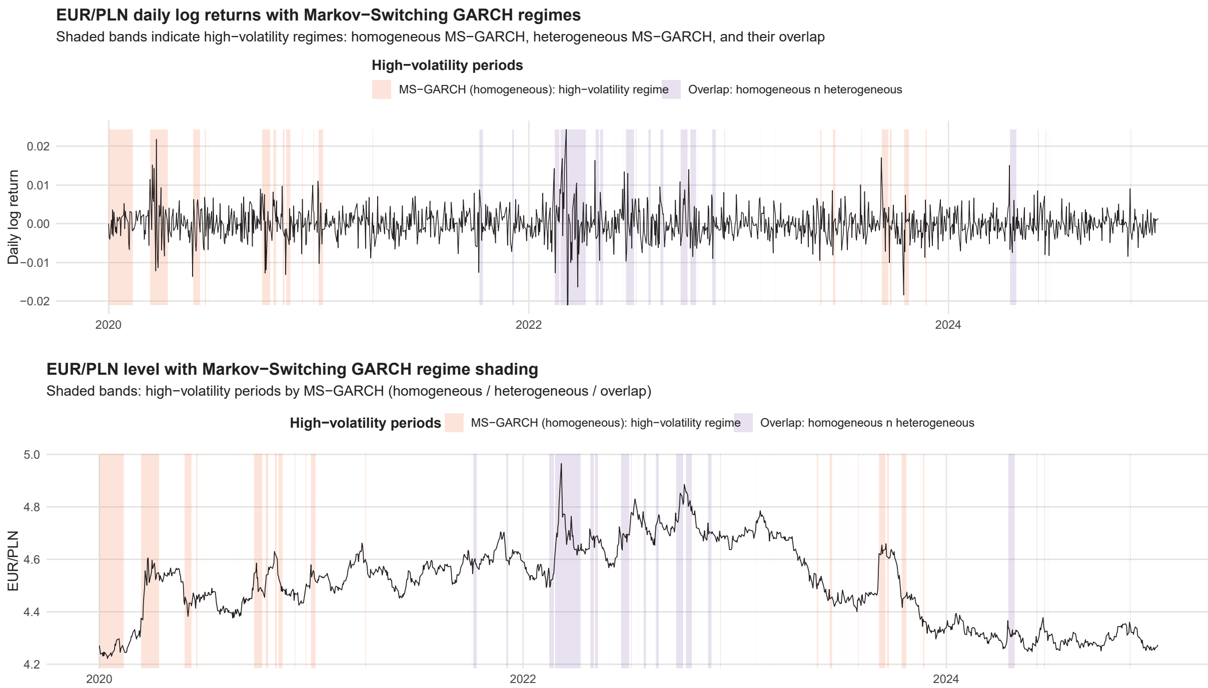

Figure 3. Periods under regimes of elevated volatility under both variants of the MS-GARCH model and the previously established 80th percentile of the volatility distribution. Visualization for log returns and the nominal EUR/PLN exchange rate

Figure 4. Forecast of the nominal EUR/PLN exchange rate volatility relative to the established parity—the ERM II central rate with a symmetric 15% deviation band. Variant of the regime: pegged exchange rate with ERM II bands

The estimated ERM II central rate for EUR/PLN was approximately 4.53. Thus, according to the developed methodological procedure, the permissible fluctuation band for the nominal EUR/PLN exchange rate within ERM II through 2028 would be [3.85, 5.20].

The second element involved developing a point forecast along with a deviation interval in the 5%–95% percentile range of the log return path using Monte Carlo simulation for both variants of the MS-GARCH model from the beginning of 2025 to the end of 2027. The visualization of the EUR/PLN volatility forecast within the ERM II fluctuation band relative to the central rate indicates, exclusively in the heterogeneous MS-GARCH model, which demonstrates higher predictive capability, a risk of exceeding the upper limit of the permissible ERM II fluctuation band. According to the results of the nominal EUR/PLN volatility forecast under the heterogeneous MS-GARCH(1,1)-std model, 21 days are subject to risk in November and December 2027.

The forecast of exchange rate volatility within the permissible deviation corridor from the ERM II central rate in figure 4 assumes the use of FX interventions, which is conditioned by a shift from a freely floating exchange rate regime to a managed float regime, a transition that also requires the transformation of the monetary policy framework. The forecast results indicate a risk of nominal EUR/PLN depreciation, which may stem from potential speculative attacks.

In the history of the European Union's institutional integration, the most well-known case was Black Wednesday (Gottschalk 2023), which involved a speculative attack and a crisis of the ERM mechanism on the British pound sterling. More recent examples of speculative attacks, such as those on the Latvian lats (Purfield and Rosenberg 2010) in response to the spillover effects of the Great Recession, and on the Croatian kuna (Brkić 2022) in March 2020 (also in the context of an economic contraction but caused by the pandemic shock), demonstrate that the risk of maintaining an exchange rate band or pegged exchange rate regime is inherently linked to speculative pressures.

Accordingly, assuming the absence of FX interventions to anchor the EUR/PLN exchange rate, and therefore the lack of intramarginal or sterilized interventions, figure 5 presents the volatility forecast and the identification of the risk of exceeding the permissible deviation corridor within the established ERM II framework.

The MS-GARCH model with homogeneous volatility regimes identified the risk of exceeding the lower ERM II band (PLN appreciation) in 353 observations between December 2026 and December 2027, while the heterogeneous MS-GARCH model identified such a risk in 252 observations between August 2026 and December 2027.

The considerations regarding both variants — under a freely floating exchange rate regime for the established ERM II corridor or a managed EUR/PLN exchange rate — are based on the assumption of systematic risk neutrality with respect to the transition of the exchange rate system,

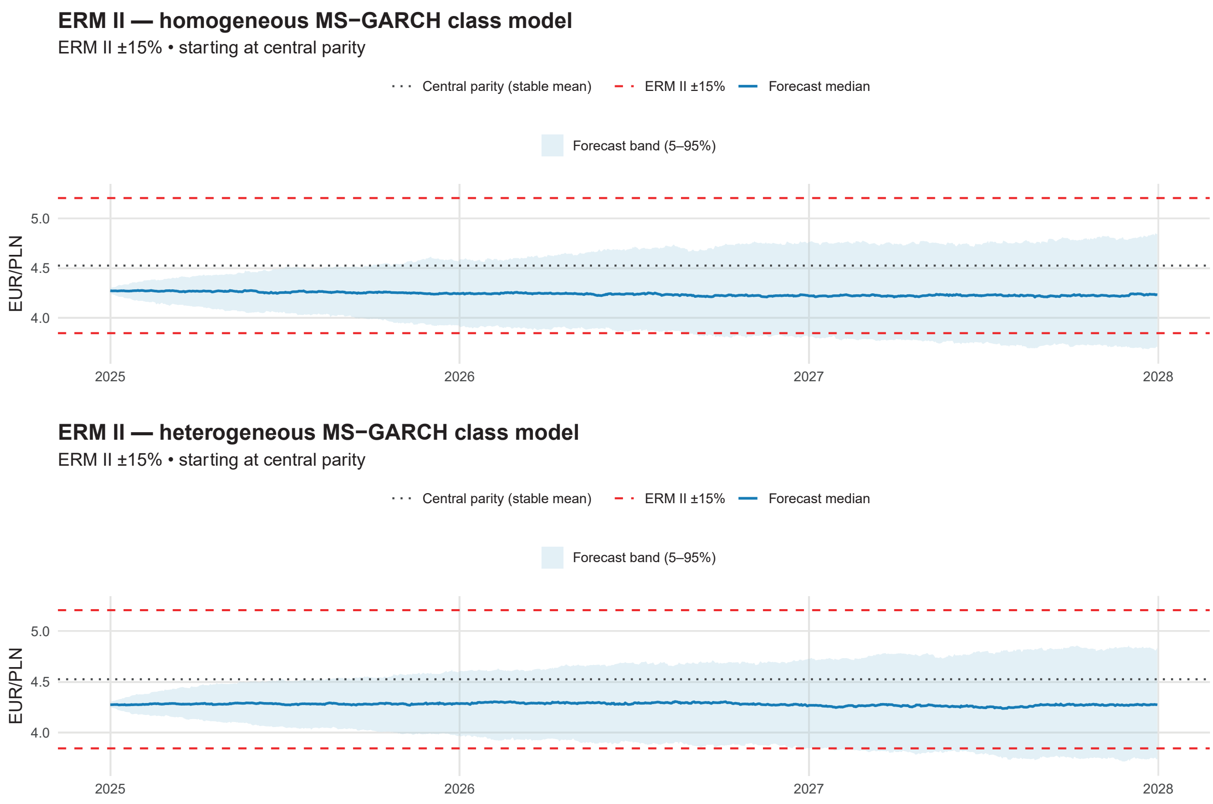

Figure 5. Forecast of the nominal EUR/PLN exchange rate volatility within the permissible ERM II deviation corridor — variant of the freely floating exchange rate regime

known as the Baxter-Stockman neutrality hypothesis (Baxter and Stockman 1989). Nevertheless, in both cases, there exists a risk of the EUR/PLN exchange rate exceeding the established symmetric deviation band from the ERM II central rate. This implies that foreign exchange interventions, particularly intramarginal interventions, are permissible during the implementation of the monetary convergence criterion under ERM II.

The research results provide a foundation for further studies on exchange rate risk analysis and volatility forecasting in the context of ERM II, as well as the identification of the optimal level of the central exchange rate. Particularly important in this respect is the inclusion of fundamental (macroeconomic) factors and structural changes that accompany the transformation of the monetary policy framework toward a pegged exchange rate band system. For this purpose, a structural VAR(CH) model under nonrecursive identification conditions for the disturbance matrix could be applied, as well as an extension toward a nonrecursive structural approach to regimes in the MS-VAR(CH) model and a Bayesian approach (Lütkepohl and Woźniak 2020). This method would allow for the incorporation not only of fundamental factors but also of conditional volatility. Alternatively, the study could be extended to the analysis of comovements with another variable using multivariate BEKK models (Hartman and Sedlak 2013) or the VECH model (Silvennoinen and Terasvirta 2008), or through their structural implementation (Fengler and Polivka 2025).

Summary

The study was based on the analysis of daily EUR/PLN exchange rate quotations for the period 2020–2024, transformed into logarithmic returns and modeled using nonlinear Markov-Switching GARCH (MS-GARCH) models in two variants: homogeneous and heterogeneous. In the first stage, the optimal structure of the GARCH(P,Q)-std model was identified through a selective-iterative procedure based on the Akaike Information Criterion (AIC), and the parameters were estimated using the quasi-maximum likelihood (QMLE) method with a conditional Student's t-distribution. Based on these results, MS-GARCH models with Markov regime switching were constructed to identify periods of low and high volatility and to estimate the conditional transition probabilities between regimes. The ERM II central parity was determined as the average exchange rate during the low-volatility regime, while the volatility forecast for 2025–2028 was generated using Monte Carlo simulations of the logarithmic return paths.

The results indicate that the optimal central parity of the EUR/PLN exchange rate, under the assumption of a symmetric ±15% fluctuation band, is approximately PLN 4.53, while the heterogeneous MS-GARCH model revealed 21 days with an elevated probability of exceeding the upper bound of the ERM II corridor in November and December 2027. These findings confirm the relevance of regime-dependent volatility modeling in anticipating exchange rate instability during potential participation in ERM II. Moreover, they empirically support the view that the dynamics of the EUR/PLN exchange rate are driven by distinct volatility regimes corresponding to periods of macroeconomic and geopolitical shocks.

Unlike equilibrium-based approaches such as FEER, BEER, or NATREX, which rely on macroeconomic fundamentals and structural long-term relationships, the proposed MS-GARCH framework provides an empirical, data-driven alternative for determining the hypothetical central parity in ERM II. This approach does not impose theoretical equilibrium constraints but instead identifies stability intervals directly from the statistical properties of the exchange rate process. As a result, it allows for a more flexible and timely assessment of nominal exchange rate stability, which is particularly valuable for emerging economies operating under a floating exchange rate regime.

The obtained results provide not only a quantitative benchmark for policymakers regarding potential ERM II entry conditions but also a methodological reference for future empirical research on forecasting exchange rate volatility and modeling FX risk. Further research should extend the analysis toward multivariate frameworks incorporating macroeconomic fundamentals, as well as cross-country comparisons with other Central and Eastern European economies in the pre-accession phase.

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Appendix

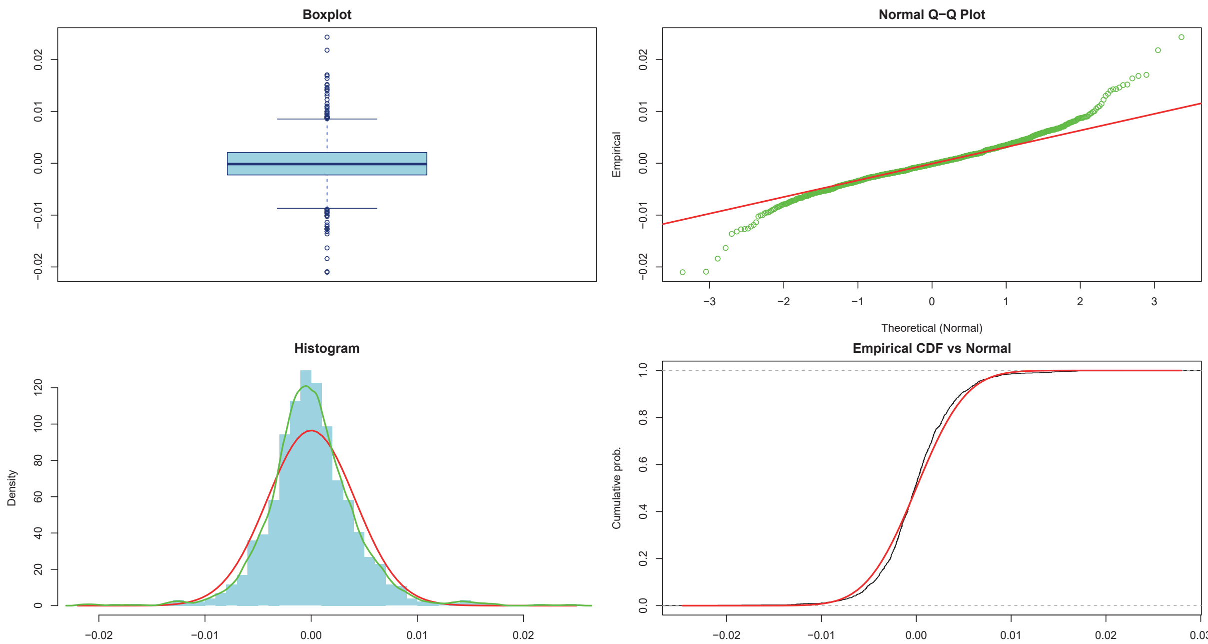

Figure 6. Visualization of the distribution of standardized residuals from the baseline regression model of logarithmic returns

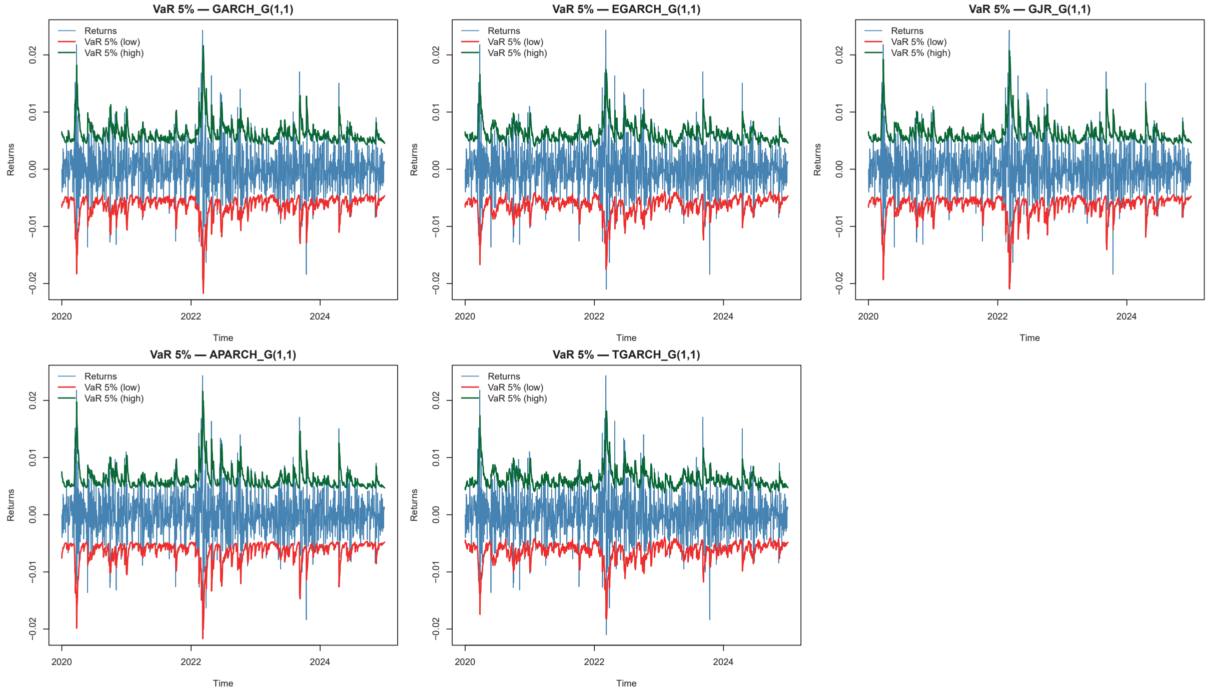

Figure 7. Calculation of the stochastic Value-at-Risk (5%) for the estimated GARCH-class models

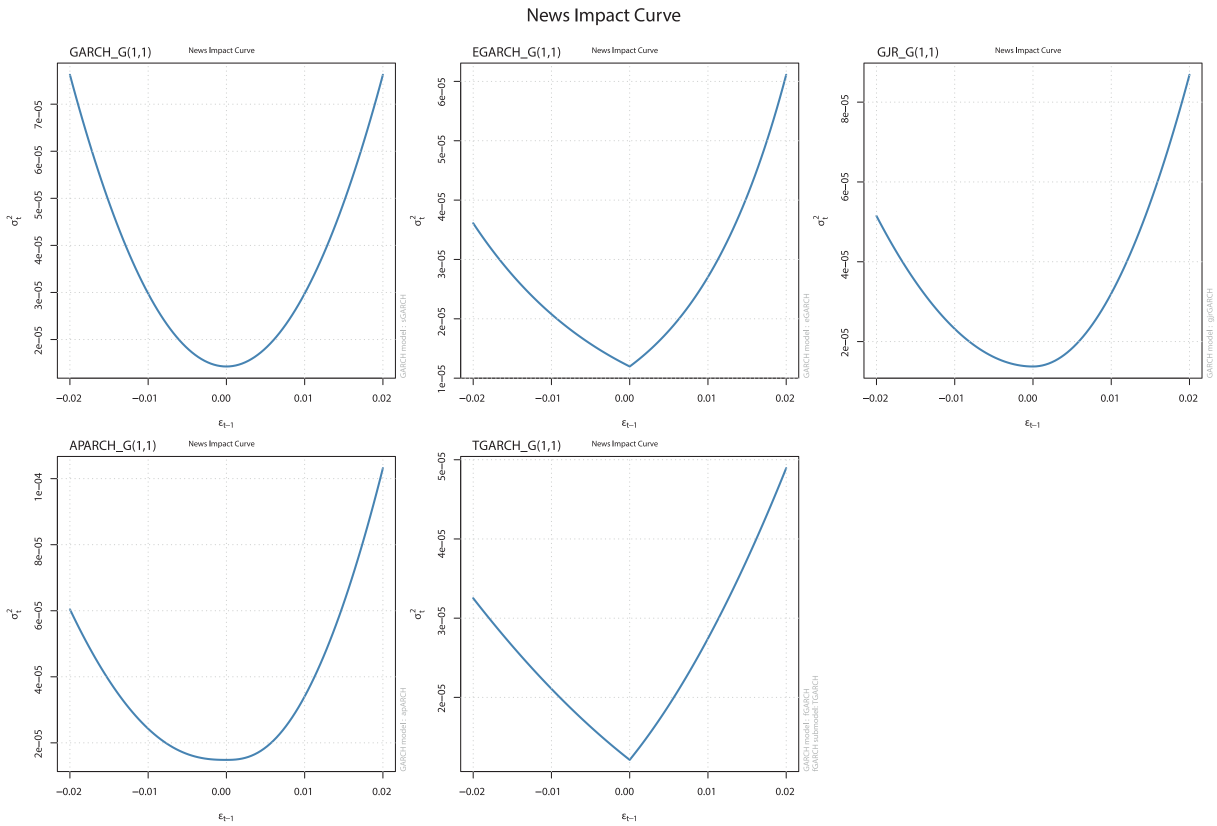

Figure 8. News Impact Curve for the estimated GARCH-class models

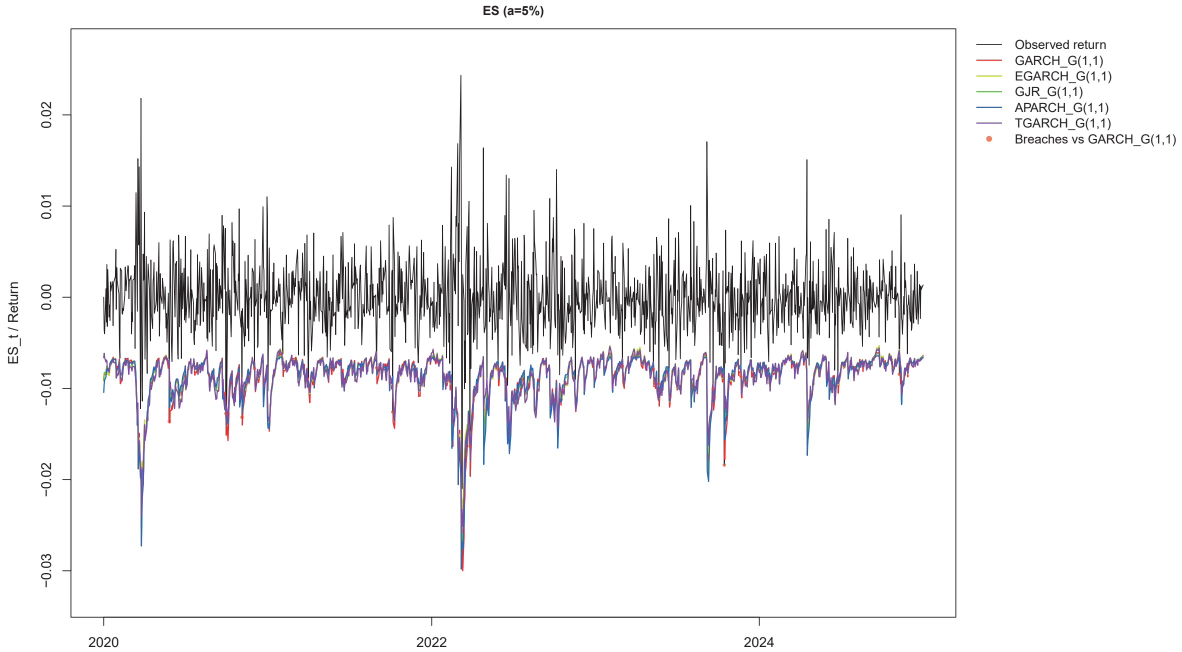

Figure 9. Expected Shortfall chart for the estimated GARCH-class models (5%) and exceedance days for the standard GARCH(1,1)-std model

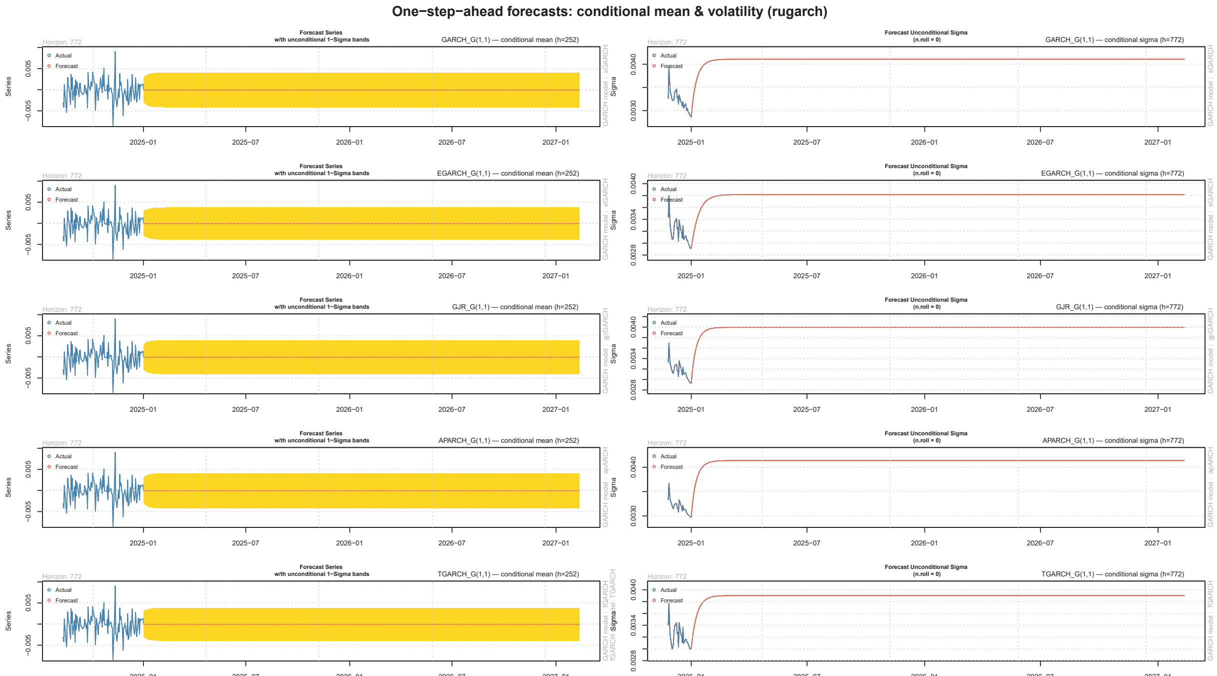

Figure 10. Forecast charts of conditional volatility and unconditional standard deviation up to 2027 under the estimated GARCH-class models