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Dynamics of Economic Development Measure. Fiftieth Anniversary of Publication of the Article by Prof. Zdzisław Hellwig

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Abstract

2018 marked the 50th anniversary of publication in Przegląd Statystyczny journal of an article entitled: "Procedure of evaluating high level manpower data and typology of countries by means of the taxonomic method" written by prof. Zdzisław Hellwig. This article is one of the most frequently quoted articles published in Przegląd Statystyczny and is one of the most often quoted articles in Polish economic literature. More importantly, the number of quotations has been growing recently. The pattern of economic development method developed in the article was the first proposal of a method of linear ordering in the area of economic research. The method is still used and has become an inspiration for many methodological studies concerning multidimensional comparative analysis written since the 1970s. Apart from drawing attention to the "phenomenon" of prof. Hellwig's publication and, in particular, its impact on the development of economic research in Poland, the main purpose of the study is presentation of the complex procedure of a dynamic approach to measuring development which integrates the issue of selection of diagnostic variables and comparability of Hellwig's development measures from different periods. The proposed procedure was illustrated with an analysis of the standard of living of residents of Polish voivodships (NUTS 2) in 2005–2016.

Keywords: Hellwig's pattern of economic development method, dynamic development measures, standard of living in Polish voivodships

JEL: C18, O11, R11

Introduction

Fifty years ago prof. Zdzisław Hellwig published in *Przegląd Statystyczny* journal an article entitled: "Zastosowanie metody taksonomicznej do typologicznego podziału krajów ze względu na poziom ich rozwoju i struktury wykwalifikowanych kadr" ["Procedure of evaluating high level manpower data and typology of countries by means of the taxonomic method"], in which he presented a number of proposals for use of a method — the so-called Wrocław taxonomy (Florek et al. 1951) in economic research (measuring differences and similarities in economic development of various countries, division of countries into more homogeneous groups and defining the structure of qualified staff corresponding to the level of economic development of the country). The method was originally developed by a group of Polish mathematicians at the University of Wrocław. The pattern of economic development method presented in the article was the first proposal of linear

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ordering in the area of economic research.[1] This method allows us to determine the ranking of objects described in the multidimensional space of features (variables), taking into account certain ordering criteria. The synthetic indicator of development, known as Hellwig's economic development measure, is used for linear ordering of objects described by many diagnostic variables, replaced by one synthetic variable. This method consists in determining the distance between the pattern which is (most frequently a non-authentic) theoretical unit that has the most favorable values for each of the variables. On the one hand, the method takes into account multidimensional aspects of social and economic phenomena and, on the other hand, man's natural desire to put the surrounding world in order, systematize it and even introduce simplifications which are so important in the process of making economic decisions.

For the last 50 years the article has been the most frequently quoted article published in *Przegląd Statystyczny* (its advantage over other articles published in this magazine has been overwhelming) as well as one of the most frequently quoted articles in Polish economic literature. More importantly, the number of quotations has been growing recently.[2]

The concept of the development measure suggested by prof. Hellwig has inspired numerous methodological studies devoted to multidimensional comparative analysis which have appeared since the 1970s. The concept was developed and used in the studies by: Cieślak (1974), Bartosiewicz (1976),

Tab. 1. Most frequently quoted articles published in *Przegląd Statystyczny* journal

Title of article	Quotations
Hellwig, Z. 1968. „Zastosowanie metody taksonomicznej do typologicznego podziału krajów ze względu na poziom ich rozwoju i struktury wykwalifikowanych kadr." No. 15 (4): 307–326	971
Chomątowski, S., and A. Sokołowski. 1978. "Taksonomia struktur." No. 25 (2): 217-226	168
Borys, T. 1978. "Metody normowania cech w statystycznych badaniach porównawczych." No. 25 (2): 371-382	146
Hellwig, Z. 1969. "Problem optymalnego wyboru predykant." No. 16 (3-4): 225-236	114
Strahl, D. 1978. "Propozycja konstrukcji miary syntetycznej. No. 25 (2): 205-215	111

Source: Google Scholar, as query result obtained with Harzing's Publish or Perish software.

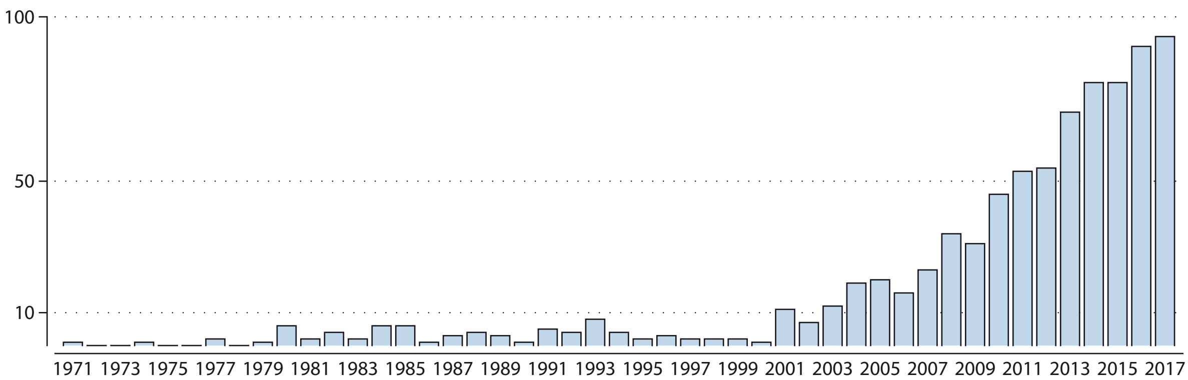


Fig. 1. Fig. 1. The number of Hellwig's article quotations in the years 1971–2017

Source: Google Scholar, as query result obtained with Harzing's Publish or Perish software.

1. Earlier, Perkal (1953) proposed a measure of linear ordering based on average standardized values of features for anthropological research.

2. We realize that the data presented on the graph is biased by varying access to scientific journals over the years. It has in recent years significantly increased the number of digitized journals, which allowed the registration of quotations. A number of articles and books published in earlier years, and especially in the 20th century, has not been digitized, which in our opinion is a signal that the number of quotations is actually greater than that given by the portal Google Scholar (see fig. 1).

Pluta (1977), Strahl (1978), Siekierka (1982), Grabiński (1984), Pociecha et al. (1988), Grabiński, Wydymus and Zeliaś (1989), Nowak (1990), Walesiak (1990), Jajuga (1993), Zeliaś (2000), Kukuła (2000), Malina (2004), Strahl (2006), Łuczak and Wysocki (2013), or Pietrzak (2014), to mention just a few. Analysis of selected articles[3] in which reference is made to prof. Hellwig's article shows that it is mostly the pattern of economic development method that is mentioned in them; the method is used by the authors to determine development measures. The article inspired both modifications and creation of new methods of linear ordering (Pociecha 2008); it also inspired the discovery of new applications for the pattern of economic development method.

Thus, the manner of determining development measures suggested by prof. Hellwig is used not only to put objects in order (countries, regions, counties, communes) as regards their economic development, but also to assess such elements as:

- quality and standard of living in regions (Janusz 2018; Murawska 2014; Ostasiewicz 2004; Zapotoczna 2014),
- cultural activity of theatres (Gałecka and Smolny 2018),
- financial condition of companies listed on a stock exchange with Taxonomic Measure of Investment Attractiveness — TMAI (Tarczyński 2001, 300–321),
- condition of natural environment (Iwacewicz-Orłowska and Sokołowska 2018),
- demographic potential (Majdzińska 2018),
- the level of smart growth of regions (Murzyn 2018),
- innovation potential of countries (Roszko-Wójtowicz and Białek 2017), regions (Majka and Jankowska 2017), enterprises (Klosa and Widera 2017), or
- the state of public safety in selected countries (Wierzbicka and Żółtaszek 2015).

Professor Hellwig emphasized a very important issue in his article — proper selection of the so-called diagnostic variables). In our opinion some authors who use the pattern of economic development method tend to attach too little importance to this issue. We believe that it is the knowledge about the analyzed phenomenon that matters most. The scientist conducting the study should suggest a set of best possible potential explanatory variables. However, formal (statistical) methods may prove very useful in making the final selection. Development measure is a latent variable, whose value becomes observable only after transformation of diagnostic variables. For example, introducing several strong correlated variables (i.e., such variables that "duplicate" information about a given phenomenon) will distort the results, making them "overdependent" on these correlated variables. Hellwig (1981) suggested a parametric variable selection method based on a correlation coefficients matrix which is used to determine the so-called clusters. Central features (variables) of clusters and isolated features are considered diagnostic variables. Over the years several other methods of formal selection of diagnostic variables based on the analysis of correlation coefficients have been suggested (Nowak 1990).

There are two approaches to development measures: static and dynamic. In the former measures are determined for the particular objects separately in each analyzed period. The latter approach consists in one-time determination of development measures on the basis of information about "object-periods." Both approaches lead to measure matrixes of identical dimensions $n \times k$ (n — number of objects, k — number of periods) but elements of these matrixes are interpreted differently. In the first case development measures provide a possibility of ordering objects along the development scale of a static character, with measures from different periods not being comparable. In the second case synthetic development measures are obtained which provide a possibility not only of ordering objects along the development scale but also of assessment of the size and directions of changes taking place during the analyzed period (Grabiński, Wydymus, and Zeliaś 1989, 93).

However, it is the static approach — i.e., ordering (classification) of sets of objects for the same (one) period that is most frequently applied in the studies which use Hellwig's development measure. And yet social and economic development is a process of positive quantitative and qualitative

3. In this part of the study only sample and not necessarily the most representative articles concerning different applications of economic development measures have been mentioned. Unfortunately, due to a large number of articles in which this method is used, the authors could not provide full bibliographic review (it was not the purpose of this article either).

changes taking place in space and time. Consequently, we may deal with a different level of development and different ordering of objects in every time unit. Obviously, having data from different periods (e.g., years) does not mean a dynamic approach. One of few examples of the dynamic approach is the study devoted to the standard of living in European countries between 1960 and 1980 carried out by Grabiński, Wydymus and Zeliaś (1989, 93–96). These authors found that dynamic measures for all countries clearly tended to increase and have a regular trend in time whereas static measures tended to vary as regards the developmental directions and their regularity in time was smaller.[4]

Apart from drawing attention to the "phenomenon" of prof. Hellwig's publication and, in particular, its impact on the development of economic research in Poland, the main purpose of the study is presentation of the complex procedure of a dynamic approach to measuring development which integrates the issue of selection of diagnostic variables and comparability of Hellwig's development measures from different periods. The proposed procedure was illustrated with an analysis of the standard of living of residents of Polish voivodships (NUTS 2) in 2005–2016.

1 Methodology

We assume that we want to analyze changes in the standard of living in n objects (i.e., regions) ($i = 1, 2, 3, ..., n$) during k periods (i.e., years) ($t = 1, 2, 3, ..., k$). In order to do that we have to select the most appropriate set of m diagnostic variables ($j = 1, 2, 3, ..., m$). Regardless of whether we use a static or dynamic approach, these variables should be the same in all periods. In the static approach we determine development measures in each of the periods separately, by setting a development pattern relevant to each period and then by determining the distances between the analyzed objects and the development pattern. We can compare changes in the standard of living of residents of i-th region in different periods by comparing changes in the places in classification in each of the periods. Measures of the standard of living in different periods cannot be compared because they were determined using different patterns of the standard of living. We believe that this means a significant loss of information. Therefore, the dynamic approach is much better. In this approach one development pattern is set for all objects and periods. The determined distances between all objects in each of the periods and the development pattern are comparable. However, several methodological problems appear here which we want to highlight.

The first thing is the selection of diagnostic variables. We believe that substantive (on the basis of content) selection based on the knowledge of the analyzed phenomenon is all-important here. Once we determine potential explanatory variables explaining the analyzed phenomenon, reduction of this set using formal statistical methods is worth carrying out. In the case of data from many periods it may turn out that slightly different sets of diagnostic variables describe the analyzed phenomenon best in each period, but we need one set of diagnostic variables for all periods. It is impossible to find a good solution here; we can only find a satisfactory one. We propose that a set of potential variables describing the analyzed phenomenon should be determined in which each observation of analyzed variable is the average of observations for all k periods; due to quite frequent lack of normal distribution of analyzed variable we propose that the median should be the average measure:

$$(1) \quad x_{ij} = \text{median}[x_{ijt}] \qquad \text{for } t = 1, 2, 3, \ldots, k \quad \text{where } i = 1, 2, 3, \ldots, n; \; j = 1, 2, 3, \ldots, m$$

where:

x_{ijt} — value of j-th potential variable explaining the analyzed phenomenon in i-th object in t period, and

x_{ij} — the median of j-th potential variable describing the analyzed phenomenon in i-th object calculated on the basis of all k periods.

The proposed formal variable selection method is based on a condition number of a correlation matrix of potential explanatory variables (Malina and Zeliaś 1996, 86). Just like in the case of average

4. In our opinion comparing static measures from different periods is unjustified and, as shown in the discussed study, such comparison does not provide interpretable results.

measure, lack of normal variable distribution is the reason why Spearman's rank correlation coefficient rather than Pearson's linear correlation coefficient is more appropriate to measure correlation.

The selection process for a diagnostic variable set proceeds as follows:

- Calculating the correlation matrix $\mathbf{R} = [r_{jl}]$ between potential explanatory variables, where r_{jl} is Spearman's rank correlation coefficient between j-th and l-th potential explanatory variables (j = 1, 2, 3, ..., m; l = 1, 2, 3, ..., m)
- Determining the inverse matrix $\mathbf{R}^{-1}$. It is worth noting that the j-th diagonal element of the $\mathbf{R}^{-1}$ matrix equals one, if the X_j variable is orthogonal in relation to remaining variables. If it is not orthogonal, then it is an element of the set (1; ∞). When the variables are too heavily correlated with each other, the diagonal elements of the inverse matrix $\mathbf{R}^{-1}$ are much larger than unity, which is a symptom of an ill conditioned matrix $\mathbf{R}$.
- Eliminating variables controlled by means of the measure of an ill conditioned matrix (Položij 1966, 349). Let us take a number N to become the measure of an ill conditioned matrix $\mathbf{R}$:

$$N = \frac{1}{m} N(\mathbf{R}) \cdot N(\mathbf{R}^{-1}). \tag{2}$$

The smaller the number N is, the better the matrix numerically conditioned becomes. $N(\mathbf{R})$ is defined as a root mean square of the elements forming the $\mathbf{R}$ matrix, becoming its norm:

$$N(\mathbf{R}) = \sqrt{\sum_{j=1}^{m} \sum_{l=1}^{m} r_{jl}^2} \tag{3}$$

The best conditioned matrixes are the orthogonal ones. In these the number conditioning the matrix equals unity. This value should be strived for, by eliminating in sequence those variables which correspond with the largest values of the diagonal elements of the $\mathbf{R}^{-1}$ matrix.

- In practice, it is difficult to attain an orthogonal matrix with more than one indicator. This is why a so-called "stop algorithm" $N(\mathbf{R})$ is subjectively established. The researcher determines a value to be small enough, and after exceeding it calculations (eliminations) are stopped.

Diagnostic variables chosen by this method become the basis for calculating of economic development measure.

The pattern of economic development method consists of five stages:

- Transforming destimulus into stimulus variables in order to standardize. The terms "stimulus" and "destimulus" variable were introduced by Hellwig (1968b, 323). The variable is a stimulus when its higher amount means a higher level of development and the variable is a destimulus when its higher amount means a lower level of development. Destimulus are most frequently transformed into stimulus using the following formulas (Kukuła 2000, 58):

$$x'_{ijt} = \frac{1}{x_{ijt}} \quad \text{for} \quad x_{ijt} > 0, \tag{4}$$

$$x'_{ijt} = a - x_{ijt} \quad \text{for} \quad x_{ijt} \leq a, \tag{5}$$

where a is constant (for variables in the form of shares $a = 1$).

- Standardization of diagnostic variables. In the case of the dynamic approach, we carry out standardization by determining arithmetic mean and standard deviation for each variable on the basis of data from all periods and all objects (Nowak 1990, 152–154):

$$\bar{x}_j = \frac{1}{n \cdot k} \sum_{i=1}^{n} \sum_{t=1}^{k} x_{ijt}, \qquad j = 1, 2, 3, \ldots, m, \tag{6}$$

$$s_j = \sqrt{\frac{1}{n \cdot k - 1} \sum_{i=1}^{n} \sum_{t=1}^{k} (x_{ijt} - \bar{x}_j)^2}, \qquad j = 1, 2, 3, \ldots, m, \tag{7}$$

$$z_{ijt} = \frac{x_{ijt} - \bar{x}_j}{s_j} \qquad \text{for } t = 1, 2, 3, \ldots, k \quad \text{where } i = 1, 2, 3, \ldots, n; \ j = 1, 2, 3, \ldots, m. \tag{8}$$

- Setting the development pattern. In the case of the dynamic approach, the development pattern is a vector whose each of $j = 1, 2, 3, ..., m$ coordinates is the maximum observed value of j-th standardized variable out of all objects in all periods:

$$z_{0j} = \max\,[x_{ijt}] \qquad \text{for } i = 1, 2, 3, \ldots, n;\ t = 1, 2, 3, \ldots, k \quad \text{where } j = 1, 2, 3, \ldots, m. \tag{9}$$

- Determining Euclidean distance between each m-dimensional object in each period and the pattern of the standard of living:

$$d_{it} = \sqrt{\sum_{j=1}^{m}(z_{ijt} - z_{0j})^2}, \qquad i = 1, 2, 3, \ldots, n;\ t = 1, 2, 3, \ldots, k, \tag{10}$$

where d_{it} distance from the development pattern of i-th object in t period.

- Determining the measure of economic development for each object in each period:

$$M_{it} = 1 - \frac{d_{it}}{\bar{d}_{0t} + 2s_d}, \qquad i = 1, 2, 3, \ldots, n;\ t = 1, 2, 3, \ldots, k, \tag{11}$$

where:

$$\bar{d}_{0t} = \frac{1}{n}\sum_{i=1}^{n} d_{it}, \qquad t = 1, 2, 3, \ldots, k, \tag{12}$$

$$s_d = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(d_{it} - \bar{d}_{0t})^2}, \qquad t = 1, 2, 3, \ldots, k. \tag{13}$$

The measure does not exceed value 1 and rarely it takes small negative value.

2 Data

The proposed procedure was illustrated with an analysis of changes in the standard of living of residents of Polish voivodships (NUTS 2) in 2005–2016. The following 11 variables were established as potential explanatory variables:

- average monthly disposable income per capita, Poland = 100
- expressways and highways per 10 000 km^2 (km)
- consumption of gas in households per year per capita (m3)
- doctors authorized per 10 000 population
- gross domestic product per capita, Poland = 100
- suicides per 10 000 population
- passenger cars per 1 000 population
- unemployment rate—on the LFS basis (%)
- infant deaths per 1 000 live births
- life expectancy of females (years)
- life expectancy of males (years)

Using the information published by the Central Statistical Office of Poland on the website devoted to Sustainable Development Indicators,[5] data for each of the specified variables in 2005–2016 were collected. As mentioned above, to generate variables further used to determine the optimal set, medians were used due to lack of normal distribution of observations across the time and presence of far outliers (see fig. 2 on pages 162–163), which both bias the value of the average (Maronna, Yohai, and Martin).

5. See: http://wskaznikizrp.stat.gov.pl/index.jsf.

3 Results

3.1 Selection of diagnostic variables

In accordance with the described methodology, the median of each potential explanatory variable in 2005–2016 was determined for each voivodship. A set of variables determined in this way was used to select diagnostic variables using a method based on a condition number of a correlation matrix of potential explanatory variables. The "stop algorithm" $N(\mathbf{R})$ was established at value 2. Finally, the set of diagnostic variables of the standard of living in Polish voivodships in 2005–2016 consists of 6 variables.

Tab. 2. Medians of potential explanatory variables in the years 2005–2016 by voivodship

Voivodship	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}
Dolnośląskie	104,23	11,22	114,60	37,58	111,20	1,83	462,60	10,85	6,20	80,25	71,90
Kujawsko-Pomorskie	90,19	5,28	62,05	28,69	82,50	1,50	451,25	10,80	5,74	79,95	71,80
Lubelskie	85,16	0,17	70,45	36,44	69,65	1,70	427,30	9,90	4,62	81,05	71,45
Lubuskie	97,71	7,43	125,30	23,94	84,35	1,93	473,00	9,55	5,75	80,00	71,50
Łódzkie	98,85	4,24	56,70	41,85	92,70	1,82	461,20	9,25	4,74	79,45	70,25
Małopolskie	94,12	6,67	122,9	35,30	88,90	1,49	440,85	9,10	4,41	81,55	73,80
Mazowieckie	128,46	3,62	144,95	46,28	158,75	1,48	507,65	7,65	4,42	81,15	72,65
Opolskie	95,78	9,36	64,30	24,27	81,60	1,33	498,70	9,35	4,80	80,55	73,05
Podkarpackie	76,62	0,00	112,75	24,09	70,70	1,54	406,35	12,05	5,27	81,90	73,90
Podlaskie	95,17	0,00	36,60	40,99	72,90	1,54	399,75	9,15	4,64	81,85	72,80
Pomorskie	105,31	6,56	92,90	36,94	96,60	1,57	455,70	8,95	4,44	80,85	73,15
Śląskie	100,75	18,41	95,30	36,03	106,05	1,13	452,05	8,85	5,88	79,75	71,75
Świętokrzyskie	86,33	3,56	61,70	27,77	76,40	1,64	430,75	12,05	4,78	81,20	71,90
Warmińsko-Mazurskie	91,92	2,67	60,90	24,61	72,20	1,51	408,10	9,75	4,98	80,55	71,40
Wielkopolskie	94,80	8,12	129,45	30,69	106,75	1,54	518,60	8,40	4,48	80,55	72,70
Zachodniopomorskie	100,23	6,63	138,10	34,50	85,00	1,81	424,90	10,65	5,30	80,25	71,55

Note: [In the journal European practice of number notation is followed—for example, 36 333,33 (European style) = 36 333.33 (Canadian style) = 36,333.33 (US and British style).—Ed.]

Tab. 3. Remaining variables at each step of variables choice procedure

Remaining potential explanatory variables	$N(\mathbf{R})$
X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11} (initial/full set)	11,429
X_1, X_2, X_3, X_4, X_6, X_7, X_8, X_9, X_{10}, X_{11}	8,534
X_1, X_2, X_3, X_4, X_6, X_7, X_8, X_9, X_{11}	6,164
X_1, X_2, X_3, X_4, X_6, X_7, X_9, X_{11}	4,012
X_2, X_3, X_4, X_6, X_7, X_9, X_{11}	3,060
X_3, X_4, X_6, X_7, X_9, X_{11}	1,726

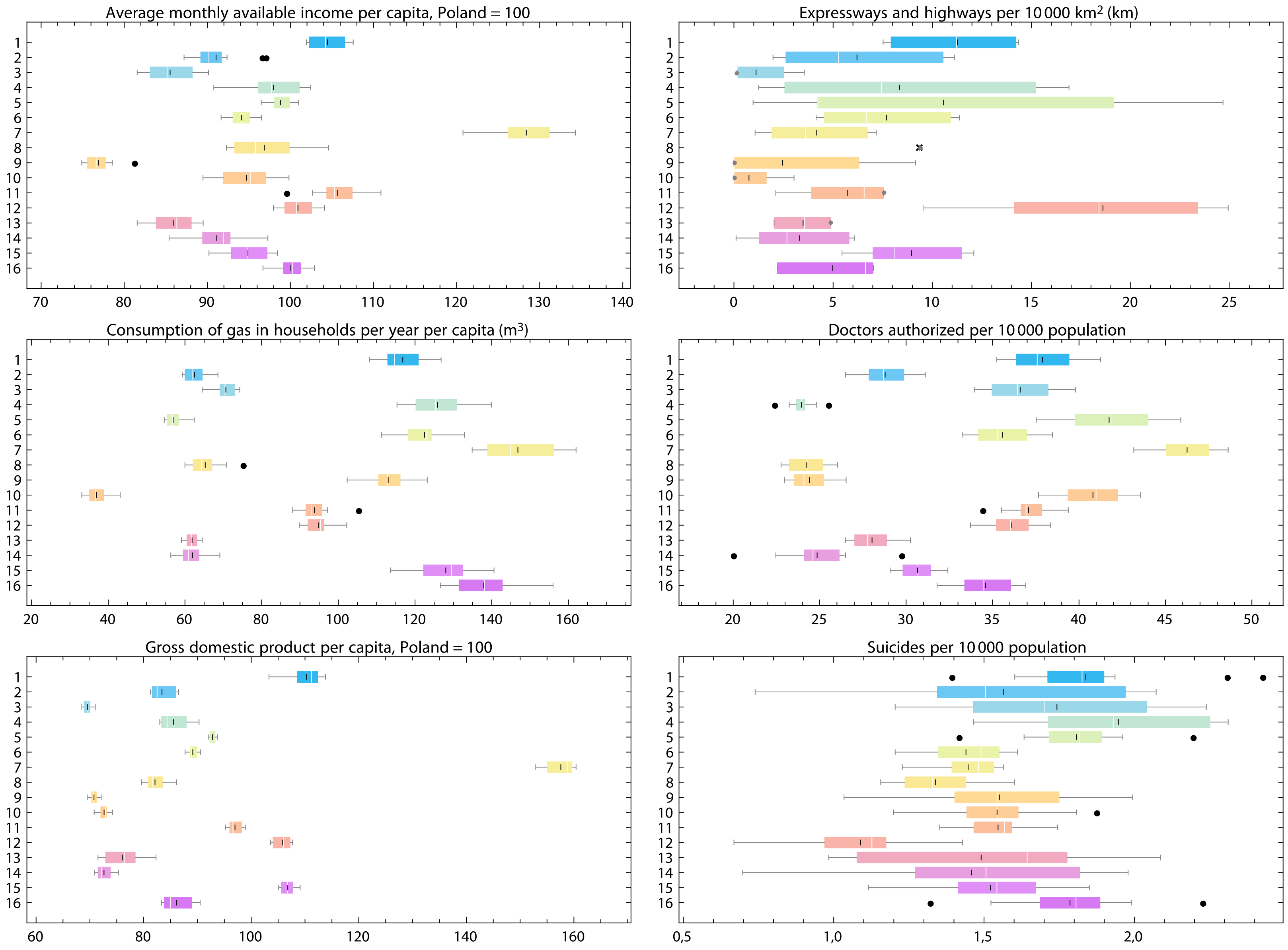
3.2 Transforming destimulus into stimulus

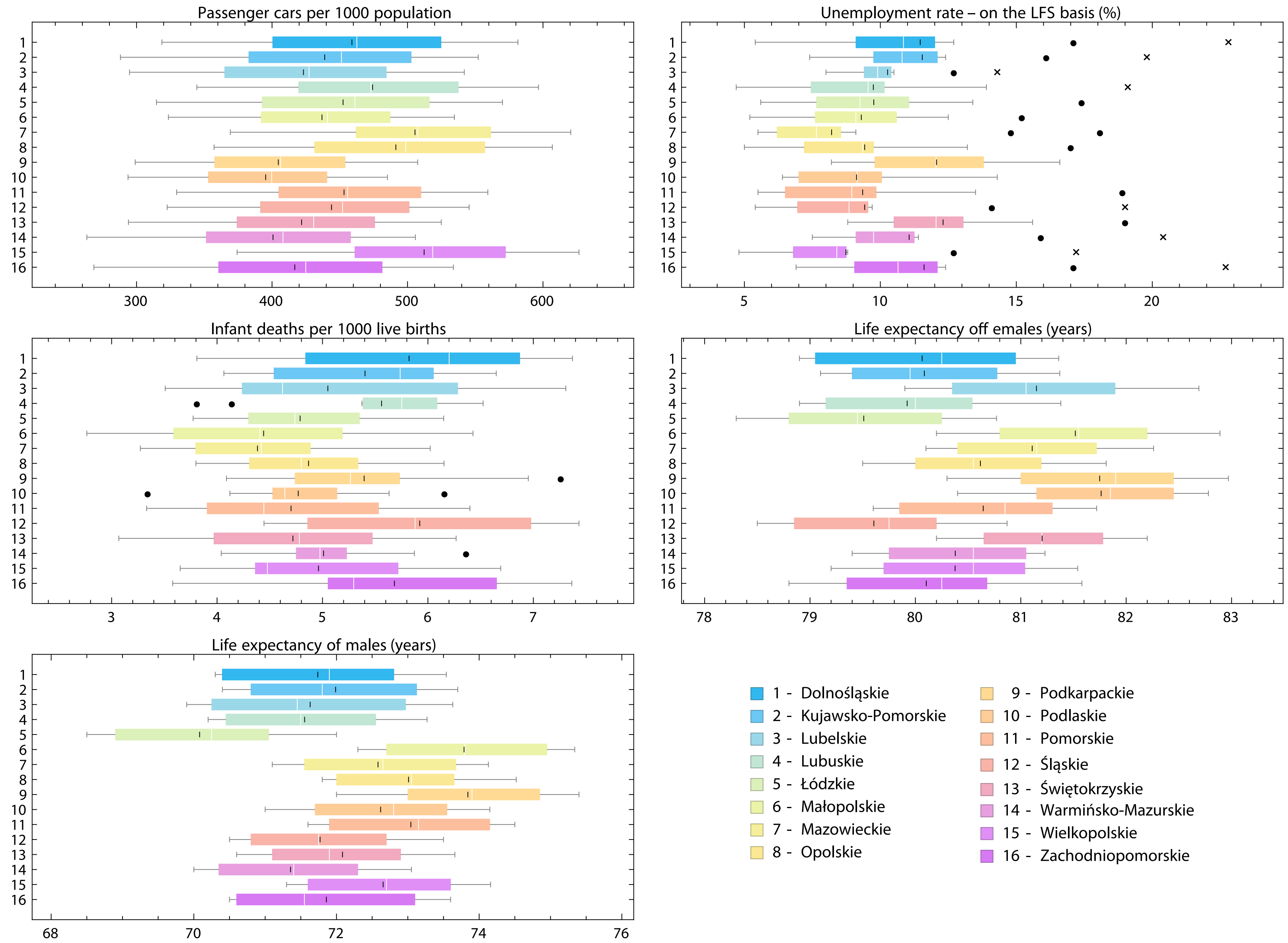
Among the selected 6 diagnostic variable destimulus are: X_6—suicides per 10 000 population, and X_9—infant deaths per 1 000 live births. Values of X_6 were converted to stimulus variable by formula $x'_{i6t} = 10\,000 - x_{i6t}$. Values of X_9 were converted by formula $x'_{i9t} = 1000 - x_{i9t}$.

3.3 Pattern of development

Out of the six diagnostic variables, five had the highest values in 2016 with only infant deaths indicator being the lowest in 2015. Two diagnostic variables were the highest in the Mazowieckie Voivodship. The other voivodships in which diagnostic variables were the highest included the Małopolskie, Podarpackie, Śląskie, and Wielkopolskie.

Fig. 2. Box plots of all potential explanatory variables




Passenger cars per 1000 population
Unemployment rate – on the LFS basis (%)
Infant deaths per 1000 live births
Life expectancy off emales (years)
Life expectancy of males (years)
1 - Dolnośląskie
2 - Kujawsko-Pomorskie
3 - Lubelskie
4 - Lubuskie
5 - Łódzkie
6 - Małopolskie
7 - Mazowieckie
8 - Opolskie
9 - Podkarpackie
10 - Podlaskie
11 - Pomorskie
12 - Śląskie
13 - Świętokrzyskie
14 - Warmińsko-Mazurskie
15 - Wielkopolskie
16 - Zachodniopomorskie

Tab. 4. Standard of living maximal (optimal) values of the variables in the years 2005–2016

Variable	Value		Year	Voivodship
	Orginal	Standardized		
X_3	161,90	2,030	2016	Mazowieckie
X_4	48,63	2,213	2016	Mazowieckie
$10\,000 - X_6$	9 999,33	2,740	2016	Śląskie
X_7	626,60	2,255	2016	Wielkopolskie
$1\,000 - X_9$	997,24	2,272	2015	Małopolskie
X_{11}	75,40	2,207	2016	Podkarpackie

3.4 Changes in the standard of living of residents of Polish voivodships in 2005–2016

Using the dynamic approach described in the methodological part of this study, values of measures of the standard of living of residents of 16 Polish voivodships in 2005–2016 were determined. During the term under analysis, the standard of living of residents of all voivodships improved. Throughout the analyzed term, the highest standard of living was observed in the Mazowieckie Voivodship, with the Małopolskie Voivodship ranking second, except for 2011 (fig. 3).

However, the highest increase in the standard of living in 2016 as compared to 2005 was observed in the Dolnośląskie Voivodship, which ranked third in 2016 (by 0,534). The smallest increase in the standard of living during the term under analysis was observed in the Opolskie Voivodship. The situation is slightly different in the regions where the standard of living is the lowest. Both in 2005 and in 2016 the standard of living was the lowest in the Warmińsko-Mazurskie Voivodship (-0,062 and 0,324 respectively); however, this voivodship was "at the end" only in 2007 and in 2010. The lowest standard of living was observed in the Łódzkie Voivodship in 2006 and in 2008–2009, in the Lubuskie in 2011 and in 2013–2015 and in the Kujawsko-Pomorskie Voivodship in 2012.

Tab. 5. Standard of living measures of Polish voivodships in the years 2005–2016

Voivodship	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	Incr.[a]
Mazowieckie	0,299	0,403	0,465	0,480	0,469	0,548	0,580	0,582	0,654	0,663	0,692	0,761	*0,462*
Małopolskie	0,270	0,343	0,304	0,410	0,405	0,458	0,492	0,500	0,560	0,607	0,648	0,682	*0,412*
Dolnośląskie	0,101	0,121	0,181	0,149	0,128	0,293	0,347	0,312	0,297	0,416	0,537	0,635	*0,534*
Wielkopolskie	0,224	0,263	0,248	0,354	0,319	0,430	0,520	0,432	0,500	0,518	0,562	0,622	*0,398*
Śląskie	0,112	0,142	0,233	0,252	0,235	0,377	0,390	0,465	0,486	0,531	0,557	0,613	*0,501*
Pomorskie	0,230	0,278	0,264	0,360	0,359	0,436	0,460	0,473	0,457	0,523	0,530	0,588	*0,358*
Zachodniopomorskie	0,034	0,143	0,086	0,261	0,119	0,288	0,266	0,383	0,388	0,424	0,553	0,559	*0,525*
Podkarpackie	0,103	0,146	0,246	0,285	0,281	0,223	0,312	0,271	0,337	0,342	0,409	0,505	*0,402*
Kujawsko-Pomorskie	0,051	0,143	0,128	0,209	0,224	0,242	0,184	0,238	0,256	0,265	0,365	0,501	*0,450*
Lubelskie	0,052	0,141	0,186	0,151	0,206	0,197	0,208	0,261	0,340	0,400	0,476	0,482	*0,430*
Łódzkie	0,037	0,067	0,130	0,143	0,083	0,247	0,247	0,258	0,275	0,347	0,356	0,474	*0,437*
Podlaskie	0,121	0,160	0,201	0,196	0,206	0,286	0,334	0,333	0,353	0,374	0,384	0,468	*0,347*
Opolskie	0,209	0,191	0,273	0,228	0,257	0,314	0,401	0,266	0,394	0,403	0,405	0,432	*0,223*
Lubuskie	0,122	0,108	0,135	0,216	0,116	0,162	0,150	0,263	0,184	0,257	0,253	0,369	*0,247*
Świętokrzyskie	0,123	0,185	0,230	0,278	0,320	0,169	0,202	0,333	0,260	0,273	0,322	0,365	*0,242*
Warmińsko-Mazurskie	−0,062	0,115	0,085	0,144	0,101	0,148	0,247	0,295	0,272	0,362	0,358	0,324	*0,386*
Mean	*0,127*	*0,184*	*0,212*	*0,257*	*0,239*	*0,301*	*0,334*	*0,354*	*0,376*	*0,419*	*0,463*	*0,524*	*0,397*
Standard deviation	*0,097*	*0,091*	*0,095*	*0,101*	*0,115*	*0,118*	*0,130*	*0,106*	*0,127*	*0,120*	*0,124*	*0,122*	–

[a] Value increase from 2005 to 2016

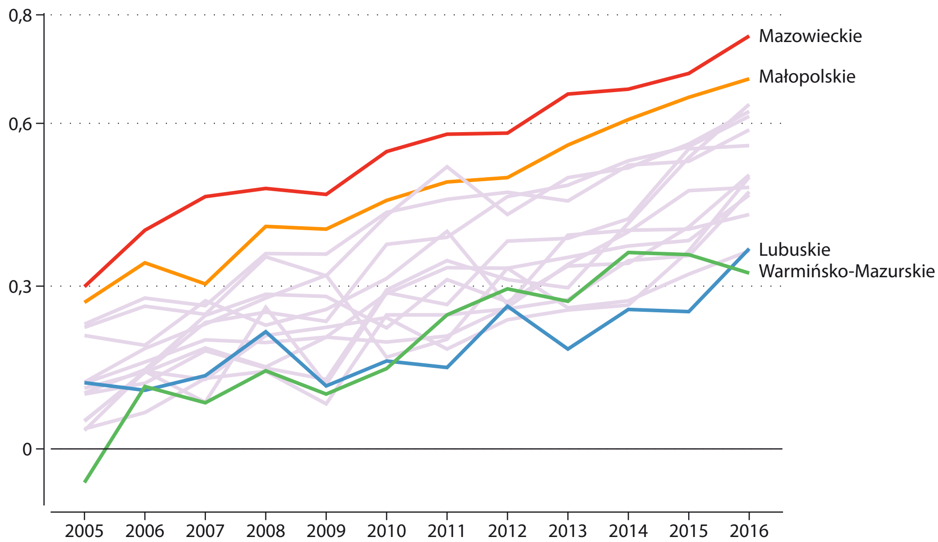


Fig. 3. Economic development measures of Polish voivodships in the years 2015–2016

Conclusions

The static approach to determining development measures proposed by prof. Hellwig is widely recognized and is still used in many studies. However, the greatest value of prof. Hellwig's paper published 50 years ago is its inspiring character which encourages further research and modifications. The dynamic approach is one such modification. This approach allows us not only to classify objects in different periods but also to compare the values of development measures in time, all the more that social-economic development is a process of positive quantitative and qualitative changes taking place in space and time.

The pattern of economic development method developed by prof. Hellwig is often called "Hellwig's method" for short, which may cause some misunderstanding. Professor Hellwig was the author of several methods, theorems and terms which have become part of econometrics as a permanent contribution of "Polish econometrics" to the development of this science, such as:

- method of optimal choice of predictors (Hellwig 1968a), also called "Hellwig's method,"
- coincidence rule (Hellwig 1976), or
- catalysis effect (Hellwig 1977).

Therefore in this case we suggest the term "Hellwig's pattern of development method."

Professor Hellwig also introduced and propagated the term "taxonometrics" analogous to such terms as econometrics, biometrics, sociometrics, or psychometrics (Hellwig 1990).

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