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Modelling and Forecasting Cash Withdrawals in the Bank

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Abstract

The goal of the paper is searching for the optimal forecasting model estimating the amount of cash withdrawn daily by the customers of one of the Polish banks by means of statistical and machine learning methods. The methodology of model creation and assessment criteria are significantly different for the models considered in the paper—i.e., for ARMAX and MLP. However, the comparison of forecasts generated by both models seems to be useful. Variables and attributes reflecting the calendar effects, used in both models, in case of obtained errors of forecasts less than 20%, showed a significant, non-linear influence of this type of predictors on the amount of the daily cash withdrawals at the bank, and hence on the amount of the daily declared cash limit.

Keywords: bank cash withdrawals forecasts, ARMAX and MLP models

Introduction

Determining the amount of money the bank has every day to ensure undisturbed cash withdrawals for its customers is an important and difficult issue. Exceeding the daily customers' demands for cash is the cost of the bank—i.e., the loss of benefits resulting from the inability to invest cash in other profitable ventures in that time. This cost is typically measured by the interest rate of interbank transactions. On the other hand, the lack of cash, even temporary, is the cost of the loss of customers' confidence in the bank, difficult to measure, leading to a serious threat. A quick transport from the Central Bank can be the answer to the lack of cash. However, it is associated with the additional cost of transport.

Some banks typically maintain as much as 40% more cash than it is needed, even though many experts consider cash excess of 15% to 20% to be sufficient (Simutis, Dilijonas, and Bastina 2008, 416). Through cash management optimization, banks can avoid falling into the trap of maintaining too much or too little cash. Therefore, it is very important to apply proper forecasting methods of cash demand. Each bank, in order to ensure the continuity of the customers' service, must determine the level of the daily limit of cash, understood as the minimal level of cash, in the bank, ensuring the continuity of the customers' service. This limit should be based on estimation of cash flows in the bank. The part of this market is determined and realized on the basis of contracts between banks. However, for the majority of banks, a significant part of their cash turnover is the customers' deposits and withdrawals. Their forecasting can be affected by a big mistake. Determining the amount of cash flows can be estimated on the basis of the knowledge of the past customers' behavior—i.e., by means of inductive reasoning.

This problem is not new. It has a wide range of research and developments around the world as well as in Poland. It is a problem of better understanding of customers' habits. Already, the first empirical studies in the 1980s made it possible, among others, to note that customers' demands for cash show sensibility to calendar effects.[1] The calendar effects are an important feature of economic data, given that most economic time series are directly or indirectly linked, to a daily activity which is usually recorded on a daily, monthly, quarterly or some other periodicity. Thus, the daily value of cash withdrawals may be influenced by daily calendar effects. The number of working days and its relation with seasonal effects are noticeable examples. Apart from the number of working days, the day of the week, the week of the month and other calendar effects such as public holidays or religious events[2] may also affect time series. Calendar effects are typically masked by strong seasonal patterns, because they are generally second-order effects only observed after other sources of variation have been accounted for (Esteves and Rodrigues 2010, 2–3). There are also important salary and pension paydays as well as locations of cash dispensers. Recently in Poland, the wide research on this issue was conducted by Gurgul and Suder (Gurgul and Suder 2013a, 2013b, 2015).

The goal of our research is searching for the optimal model estimating the amount of cash withdrawn daily by the customers of one of the Polish banks by means of statistical and machine learning methods.

1 Methodology

In the literature, techniques used for cash demand forecasting can be broadly classified into four groups (Darwish 2013, 405–406):

- time series methods that predict future cash need based on the past values of variable and/or past errors
- a factor analysis method, which is based on the determination of various factors that influence the cash demand pattern and calculating their correlation with actual cash withdrawals; in this group, the econometric models can be included
- a fuzzy expert system approach that tries to imitate the reasoning of a human operator; the idea is to reduce the analogical thinking behind the intuitive forecasting to formal steps of logic
- a neural networks approach that maps the relationships between various factors affecting the cash withdrawals and the actual cash withdrawals

In our research, we used methods of time series analysis and econometric modeling as well as artificial neural networks belonging to machine learning methods.

Experiments were performed on time series representing bank customers' cash withdrawals including 461 samples covering the period July 2012 to April 2014. The samples were not recorded on Saturdays, Sundays and Holidays, when the bank was closed. The database was divided into 5 subsets: one training data set (used to build the models) and 4 testing data sets (corresponding to the periods of forecasting).

Tab. 1. Database specification

Symbol	Period	Number of samples	Subset description
U	2012.07.02 – 2013.12.31	378	Training
T1	2014.01.02 – 2014.01.31	21	Testing
T2	2014.02.03 – 2014.02.28	20	Testing
T3	2014.03.03 – 2014.03.31	21	Testing
T4	2014.04.01 – 2014.04.30	21	Testing
BI	2012.07.02 – 2014.04.30	461	The whole data base including time series of cash withdrawals

1. Calendar effects (public and religious holidays, paydays) can significantly disturb the processes described by means of the ARIMA and SARIMA models. The first research on the influence of calendar effects on economic and financial phenomena was carried out by Cleveland and Devlin (1980) as well as Liu (1980).

2. For example, the Ramadan effect (Lee, Suhartono, and Hamzah 2010).

As predicted (dependent) variables, we used daily withdrawals in PLN (Y) as well as the natural logarithm of daily withdrawals (lnY).[3]

Our analysis of withdrawal distributions in the considered periods enabled us to distinguish the following predictors (independent variables), identified as descriptive attributes in machine learning methods: DW—the day of the week, DM—the day of the month, MY—the month of the year, TEN—the 10th day of the month (in case of the holiday, the weekday before). We started experiments with the analysis of the training time series. The shapes of withdrawal were determined with respect to the specified calendar variables. Next, the best model—i.e., the ARIMA model (Brockwell and Davis 2002; Cottrell and Lucchetti „Jack" 2015) was selected on the basis of the correlation and partial correlation functions and the AIC criterion. The analysis allowed us to formulate the final form of the ARMAX model including dependencies, estimated earlier, according to calendar variables, the artificial binary variable (TEN) delivering information about significantly larger withdrawals around the 10th day of the month, and the autoregression component AR(1) on account of autocorrelation of a random component.

As machine learning models for forecasting the customers' withdrawals, we used artificial neural networks in the form of the two and three perceptron layers (Rumelhart and McClelland 1986). Such models were trained using the backpropagation method (Fausett 1994). We assumed that all predictors are of symbolical type. Ten regressive models were created on the basis of 50% of the samples randomly selected from the training set (U). 25% of the samples were used as a validation set (stop criterion). The assessment of quality of forecasts was made by means of the following factors:

Mean Error $$\mathrm{ME} = \frac{1}{n}\sum_{t=1}^{n} e_t,$$

Mean-Square Error $$\mathrm{MSE} = \frac{1}{n}\sum_{t=1}^{n} e_t^2,$$

Root-Mean-Square Error $$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n} e_t^2},$$

Mean-Absolute-Error $$\mathrm{MAE} = \frac{1}{n}\sum_{t=1}^{n} |e_t|,$$

Mean-Percentage Error $$\mathrm{MPE} = \frac{1}{n}\sum_{t=1}^{n} 100\frac{e_t}{y_t},$$

Mean-Absolute-Percentage Error $$\mathrm{MAPE} = \frac{1}{n}\sum_{t=1}^{n} 100\frac{|e_t|}{y_t}.$$

Theil's coefficient U provides a measure of forecast precision.[4] The more accurate the forecast is, the lower the value of U is. U has a minimum of 0. This measure can be interpreted as the ratio of the RMSE of the proposed forecasting model to the RMSE of a naïve model which simply predicts for each t. The naïve model yields $U = 1$; values less than 1 indicate an improvement relative to this benchmark and values greater than 1 a deterioration. Theil's U is defined as follows:

$$U = \sqrt{\frac{1}{n}\sum_{t=1}^{n-1}\left(\frac{y^*_{t+1} - y_{t+1}}{y_t}\right)^2 \Big/ \frac{1}{n}\sum_{t=1}^{n-1}\left(\frac{y_{t+1} - y_t}{y_t}\right)^2}.$$

3. In financial research, the logarithm enables us to reduce the influence of the so-called outliers on the analyzed phenomenon.

4. This statistic, developed by Theil in 1966, is sometimes called U2, to distinguish it from a related but different U (or U1), defined in an earlier work by Theil in 1961, which is bound between 0 and 1, with values closer to 0 indicating greater forecasting accuracy. It seems to be generally accepted that the later version of Theil's U is a superior statistic.

2 Results

2.1 Forecasting by means of ARMA and ARMAX

Experiments mentioned in the section describing the methodology were carried out for both the variable Y and the variable lnY. In case of the ARMAX model, slightly better results were obtained for lnY. Therefore, in the paper, we describe only those results.[5] All calculations were done with GRETL program (Cottrell and Lucchetti "Jack" 2015; Kufel 2011).

Tab. 2. Summary statistics, using the observations 2012.07.02–2013.12.31 for the var. lnY (378 valid observations)

Statistics	Value	Statistics	Value	Statistics	Value
Mean	12,667	Std. Dev.	0,445	5% Perc.	12,044
Median	12,630	C.V. (%)	3,514	95% Perc.	13,467
Minimum	11,480	Skewness	0,653	IQ range	0,503
Maximum	14,077	Ex. kurtosis	0,974	Missing obs.	0

Note: In the journal European practice of number notation is followed — for example, 36 333,33 (European style) = 36 333.33 (Canadian style) = 36,333.33 (US and British style). — Ed.]

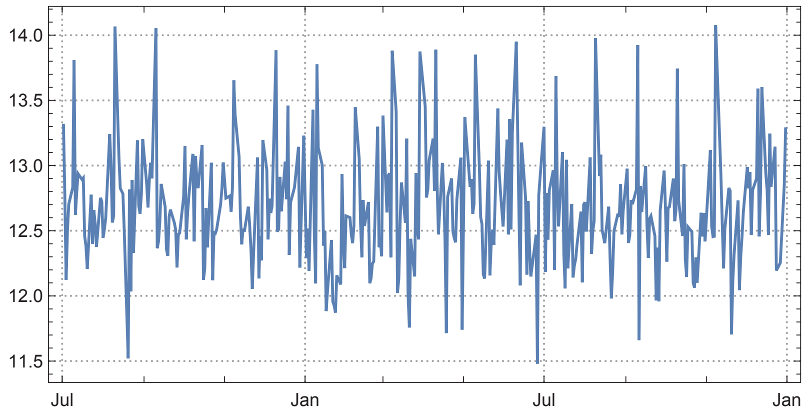

Fig. 1. The natural logarithm of daily withdrawals

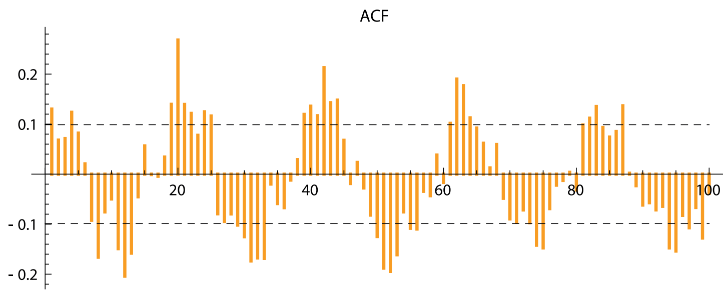

Fig. 2. Autocorrelation of the lnY time series

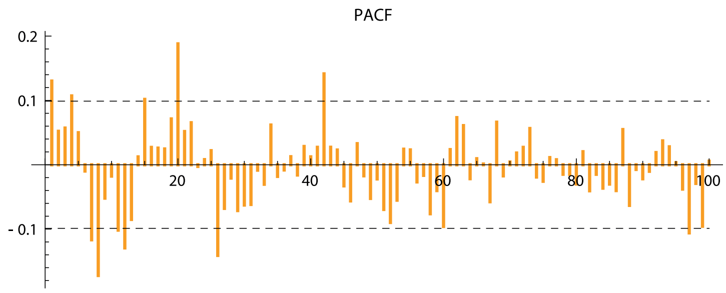

Fig. 3. Partial autocorrelation of the lnY time series]

5. The authors can make accessible models for the variable Y.

Distribution of lnY is right skewed and more slender than the normal distribution. However, both levels of skewness and kurtosis can be considered as low, which makes the distribution only slightly deviating from the normal distribution. This feature of the dependent variable allows you to use the OLS method for models estimating it.

On the basis of correlation and partial correlation functions, we decided to select the model from the family of the ARIMA models with the maximal delay equal to 30 for the AR and MA components. According to the AIC criterion, the best model—i.e., ARMA(20,1), was selected.

Tab. 3. The best candidates for modelling the lnY time series

No	Candidate	AIC
1	ARMAProcess (20,1)	−663,914
2	ARProcess(20)	−663,886
3	ARMAProcess(21,1)	−662,994
4	ARMAProcess(27,1)	−662,985
5	ARProcess(21)	−662,809
6	ARMAProcess(22,1)	−662,809
7	ARProcess(26)	−662,721
8	ARProcess(22)	−662,618
9	ARProcess(27)	−662,533
10	ARMAProcess(20,2)	−662,231

However, forecasts on the basis of the ARMA(20,1) model were inaccurate. Therefore, better models were searched for. For this purpose, models of trends of lnY with respect to the specified calendar variables were created.

The first created model concerned the trend of lnY with respect to days of the week (DW).

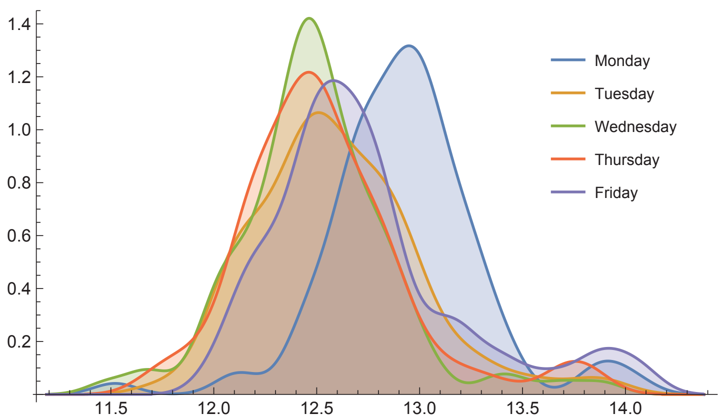

Fig. 4. Smooth histograms of lnY by weekday

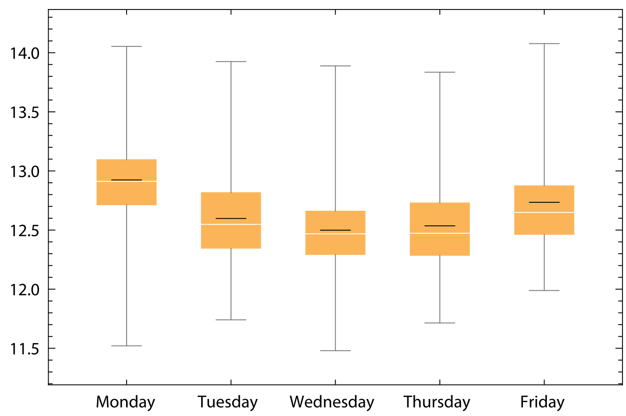

Fig. 5. Boxplots of lnY by weekday

The Kruskal-Wallis's test showed ($p = 8{,}9 \cdot 10^{-18}$) that there is a statistically significant difference among withdrawals in individual days of the week. The estimated models showed that the square trend is the best description of lnY with respect to days of the week. The minimal level of withdrawals was encountered on the third day of the week.

Tab. 4. OLS, using observations 2012.07.02–2013.12.31 (n = 378). Dependent variable: lnY. Independent variable: DW. HAC standard errors, bandwidth 5 (Bartlett kernel)

Variable	Coefficient	t-ratio	p-value
const	13,3599	157,8496	$< 0{,}0001$
DW	−0,5365	−8,2886	$< 0{,}0001$
DW^2	0,0831	7,6147	$< 0{,}0001$

Statistics	Value	Statistics	Value
Mean dependent var	12,6666	S.D. dependent var	0,4451
Sum squared resid	66,2952	S.E. of regression	0,4204
R-squared	0,1126	Adjusted R-squared	0,1078
F(2, 375)	37,8000	P-value(F)	$< 0{,}0001$
Log-likelihood	−207,3521	Akaike criterion	420,7042
Schwarz criterion	432,5089	Hannan-Quinn	425,3893
rho	0,1253	Durbin-Watson	1,7419

Note: Number "2" (as in upper index at DW variable name) denotes the exponent

The second created model concerned the trend of lnY with respect to days of the month (DM).

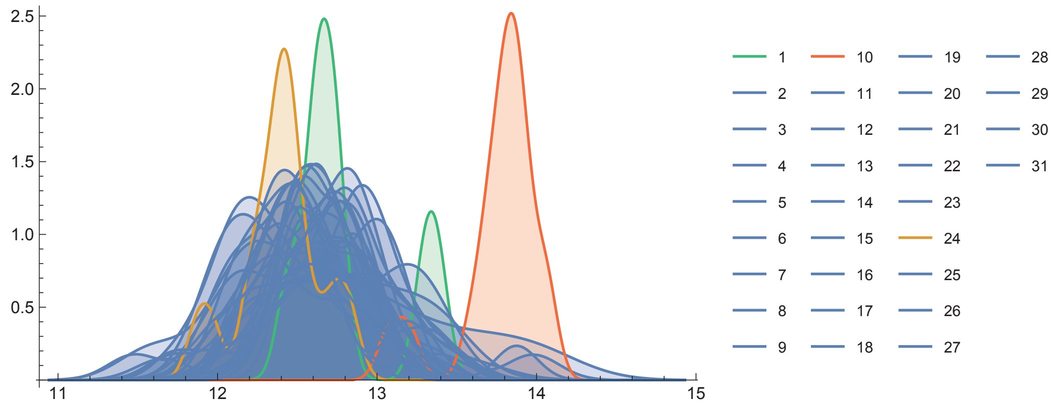

Fig. 6. Smooth histograms of lnY by day of the month

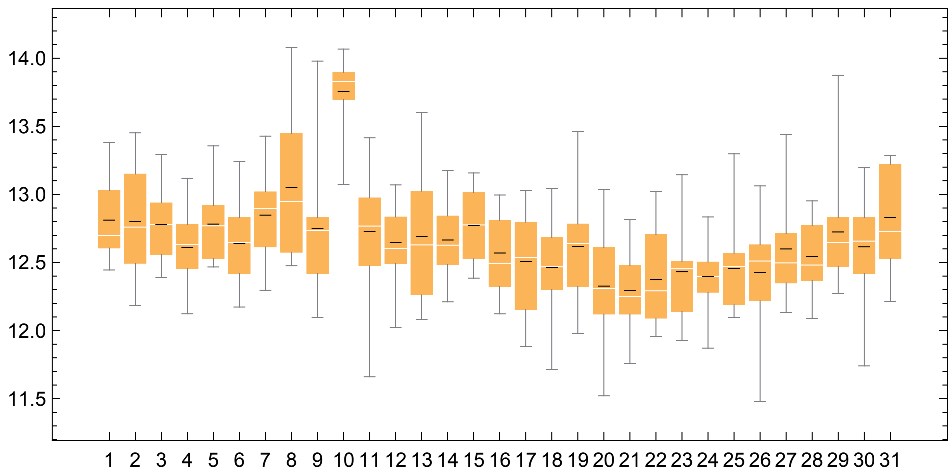

Fig. 7. Boxpplots of lnY by day of the month

The Kruskal-Wallis's test showed ($p = 4{,}1 \cdot 10^{-17}$) that there is a statistically significant difference among withdrawals in individual days of the month. The estimated models showed that the cube trend is the best description of lnY with respect to days of the month. The maximal level of withdrawals (according to the polynomial model)[6] was encountered on the 7th day of the month, whereas the minimal level was encountered on the 23rd day of the month.

Tab. 5. OLS, using observations 2012.07.02–2013.12.31 (n = 378). Dependent variable: lnY. Independent variable: DM. HAC standard errors, bandwidth 5 (Bartlett kernel)

Variable	Coefficient	*t*-ratio	*p*-value
const	12,4654	136,3660	< 0,0001
DM	0,1378	5,5685	< 0,0001
DM^2	−0,0121	−6,7795	< 0,0001
DM^3	0,0002	7,0604	< 0,0001

Statistics	Value	Statistics	Value
Mean dependent var	12,6666	S.D. dependent var	0,4451
Sum squared resid	61,5254	S.E. of regression	0,4055
R-squared	0,1764	Adjusted R-squared	0,1698
F(2, 375)	33,2711	P-value(F)	< 0,0001
Log-likelihood	−193,2399	Akaike criterion	394,4798
Schwarz criterion	410,2194	Hannan-Quinn	400,7266
rho	−0,0520	Durbin-Watson	2,0950

Note: Numbers "2" and "3" (as in upper index at DM variable name) denote the exponent

The third created model concerned the trend of lnY with respect to months of the year (MY).

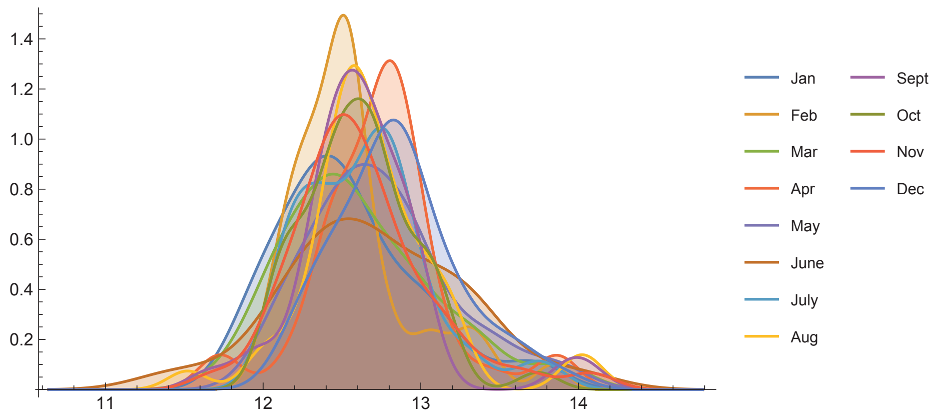

Fig. 8. Smooth histograms of lnY by month

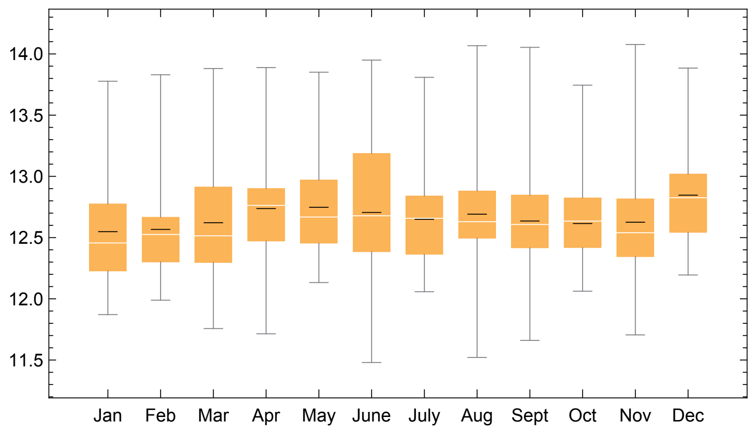

Fig. 9. Boxplots of lnY by month

6. This model does not take into consideration outliers of the 10th day of the month.

The Kruskal-Wallis's test showed ($p = 0{,}0158$) that there is a statistically significant difference among withdrawals in individual months of the year. The estimated models showed that the polynomial trend of the fifth degree is the best description of lnY with respect to months of the year.[7]

Tab. 6. OLS, using observations 2012.07.02–2013.12.31 (n = 378). Dependent variable: lnY. Independent variable: MY. HAC standard errors, bandwidth 5 (Bartlett kernel)

Variable	Coefficient	*t*-ratio	*p*-value
const	12,9125	244,0132	< 0,0001
MY^2	−0,0026	−6,3751	< 0,0001
MY^3	$7{,}64 \cdot 10^{-05}$	5,6670	< 0,0001
MY^5	$4{,}73 \cdot 10^{-07}$	1,6969	0,0905

Statistics	Value	Statistics	Value
Mean dependent var	12,6666	S.D. dependent var	0,4451
Sum squared resid	65,2489	S.E. of regression	0,4176
R-squared	0,1266	Adjusted R-squared	0,1196
F(2, 375)	21,2846	P-value(F)	< 0,0001
Log-likelihood	−204,3456	Akaike criterion	416,6911
Schwarz criterion	432,4307	Hannan-Quinn	422,9379
rho	0,0155	Durbin-Watson	1,9628

Note: Numbers "2", "3" anf "5" (as in upper index at MY variable name) denote the exponent

The estimations described earlier enabled us to specify finally the form of the ARMAX model (see more about this approach in: Bielak 2010, 39) including the autoregression component AR(1),[8]

Tab. 7. ARMAX, using observations 2012.07.02–2013.12.31 (n = 378). Dependent variable: lnY. Standard errors based on Hessian

Variable	Coefficient	*t*-ratio	*p*-value
const	13,083	141,0136	< 0,0001
phi_1	−0,1046	−2,0296	0,0424
DM	0,0702	9,9950	< 0,0001
DM^2	−0,0071	−29,2615	< 0,0001
DM^3	0,0001	48,4005	< 0,0001
TEN	1,01175	13,4056	< 0,0001
DW	−0,4914	−8,8577	< 0,0001
DW^2	0,0746	8,2255	< 0,0001
MY^2	0,0254	13,0620	< 0,0001
MY^3	−0,0035	−22,6483	< 0,0001
MY^5	$1{,}14 \cdot 10^{-05}$	33,7207	< 0,0001

Statistics	Value	Statistics	Value
Mean dependent var	12,66666	S.D. dependent var	0,44515
Mean of innovations	0,00008	S.D. of innovations	0,30351
Log-likelihood	−85,66771	Akaike criterion	195,33540
Schwarz criterion	242,55420	Hannan-Quinn	214,07580

	Real	Imaginary	Modulus	Frequency
AR Root 1	−9,5573	0,0000	9,5573	0,5000

Note: Numbers in upper index at variable names denote the exponent

7. Where parameters for MY and MY^4 are equal to 0.
8. According to autocorrelation of the error term component.

the artificial binary variable (TEN) delivering information about significantly larger withdrawals around the 10th day of the month as well as variables describing square distribution of withdrawals with respect to the days of the week (DW, DW^2), cube distribution of withdrawals with respect to the days of the month (DM^2, DM^3, DM^5) and polynomial distribution of the fifth degree of withdrawals with respect to the months of the year (MY^2, MY^3, MY^5) (see tab. 7). The proposed model satisfies basic formal criteria to acknowledge it as a credible model for forecasting. The forecasts for the first four months of 2014 have the Mean Absolute Percentage Error from 12,873 for T4 to 24,188 for T3.

Tab. 8. Errors of forecasts for lnY

	Forecasted period				
	T1	**T2**	**T3**	**T4**	**T1+T2+T3+T4**
Mean Error	0,155	0,009	−0,117	0,041	0,022
Mean Squared Error	0,071	0,050	0,072	0,027	0,055
Root Mean Squared Error	0,268	0,225	0,269	0,166	0,236
Mean Absolute Error	0,217	0,189	0,209	0,132	0,187
Mean Percentage Error	1,202	0,037	−0,958	0,313	0,150
Mean Absolute Percentage Error	1,705	1,507	1,671	1,042	1,481
Theil's U (in percentage)	0,605	0,401	0,704	0,395	0,514
Bias proportion, UM	0,336	0,001	0,190	0,063	0,008
Regression proportion, UR	0,007	0,002	0,119	0,000	0,028
Disturbance proportion, UD	0,656	0,996	0,689	0,936	0,962

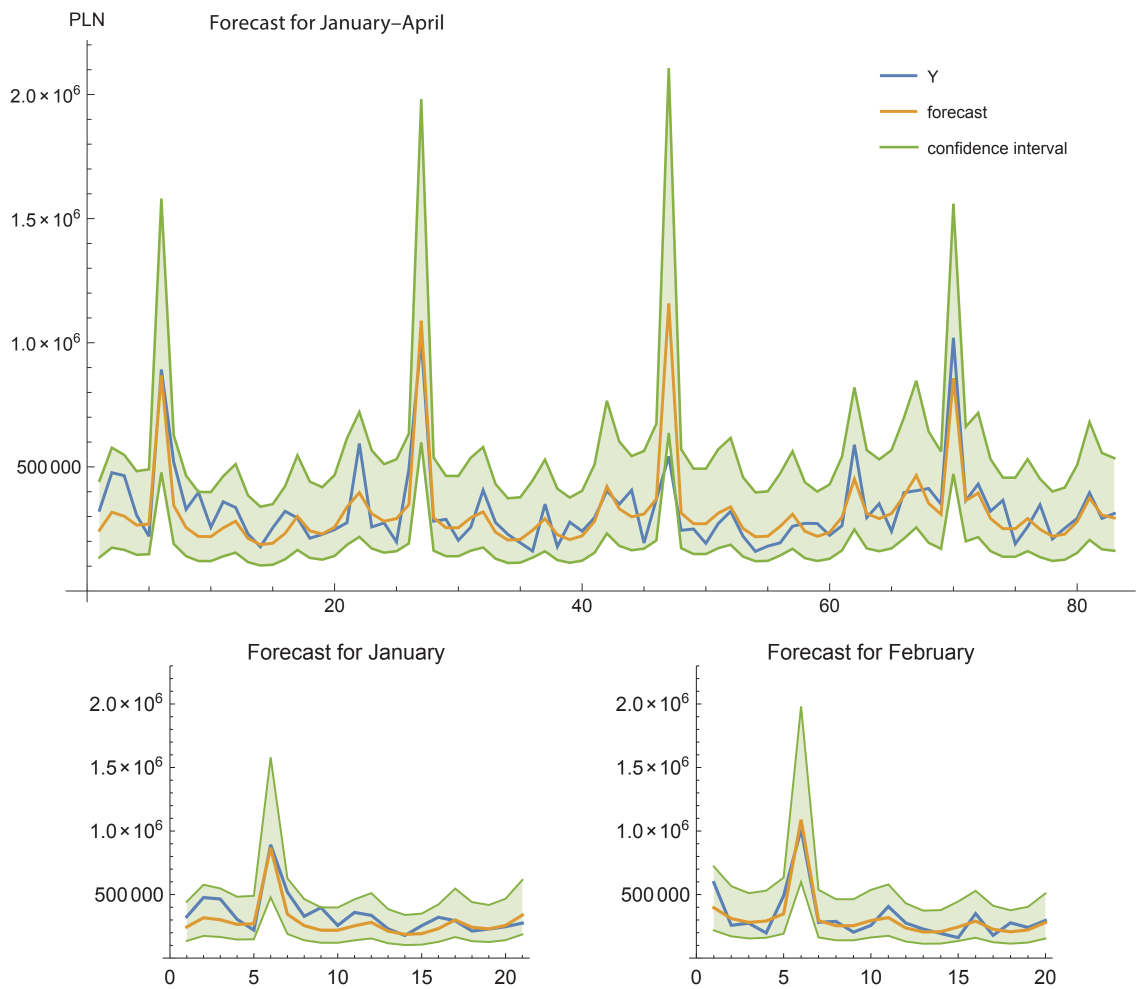

Fig. 10. Forecast of withdrawals (Y) by means of ARMAX model

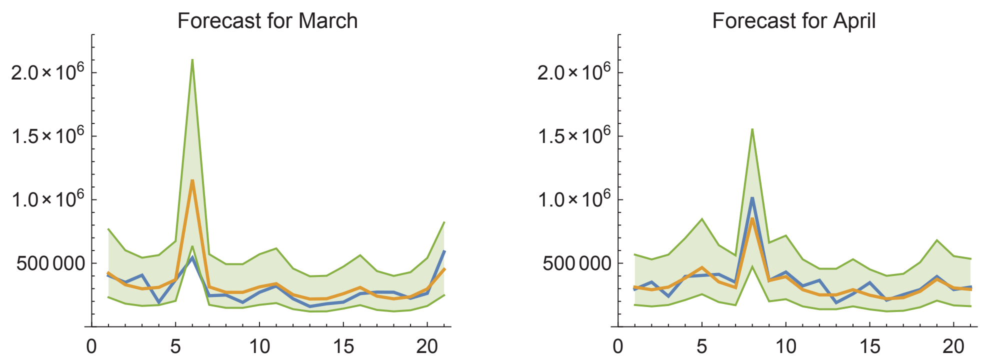

Fig. 10. (continued) Forecasts by means of ARMAX model

2.2 Forecasting by means of machine learning models

Artificial neural networks in the form of multilayer perceptron (MLP) were generated using the Data Miner package available in Statistica 7.1. Models are assessed using the regression statistics as follows (StatSoft Inc. 2013):

- Data Mean — the average value of the target output variable
- Data S.D. — standard deviation of the target output variable
- Error Mean — the average error (residual between target and actual output values) of the output variable
- Abs. E. Mean — average absolute error (the difference between target and actual output values) of the output variable
- Error S.D. — standard deviation of errors for the output variable
- S.D. Ratio — the error/data standard deviation ratio
- Correlation — the standard Pearson-R correlation coefficient between the predicted and observed output values.

The experiments mentioned in the section describing the methodology were carried out for both the attribute Y and the attribute lnY. Slightly better results were obtained for the attribute Y. Therefore, in the paper, we describe only those results. The assumed methodology and parameters for training models led us to ten models described in table 9.

Tab. 9. Artificial neural networks for withdrawals forecasting

No	Symbol	S.D. Ratio Training	S.D. Ratio Verification	S.D. Ratio Testing	Input	Hidden(1)	Hidden(2)
1	MLP 5:42-12-2-1:1	0,650379	0,706761	0,503473	5	12	2
2	MLP 5:35-12-5-1:1	0,622421	0,708558	0,547025	5	12	5
3	MLP 6:104-7-1:1	0,556769	0,705015	0,553301	6	7	0
4	MLP 3:37-5-9-1:1	0,563914	0,702839	0,555440	3	5	9
5	MLP 3:37-1-1:1	0,566053	0,705031	0,559865	3	1	0
6	MLP 3:11-12-2-1:1	0,667916	0,701674	0,568242	3	12	2
7	MLP 3:37-1-1:1	0,595329	0,704290	0,573429	3	1	0
8	MLP 4:68-12-1-1:1	0,520211	0,707218	0,578689	4	12	1
9	MLP 6:104-12-3-1:1	0,615884	0,691878	0,635807	6	12	3
10	MLP 3:11-1-1:1	0,705070	0,699969	0,686923	3	1	0

The best model of MLP 5:42–12–2-1:1 (5 input neurons, 12 neurons in the first hidden layer, 2 neurons in the second hidden layer and 1 output neuron) was selected on the basis of the S.D. Ratio estimated using a testing set that included 25 percent of samples from the set U. The iput attributes of the selected model were as follows: DW, TEN, MY, MY2, MY5. The regression statistics for the neural network are shown in Table 10, whereas the model in Figure 10. In Figure 11, time series of real and forecasted withdrawals obtained by means of the selected MLP model are compared.

Tab. 10. The regression statistics of the neural network

Regression statistics	Value
Data Mean	371190,6
Data S.D.	221434,6
Error Mean	−3034,5
Abs. E. Mean	111486,3
Error S.D.	75548,0
S.D. Ratio	0,5
Correlation	0,9

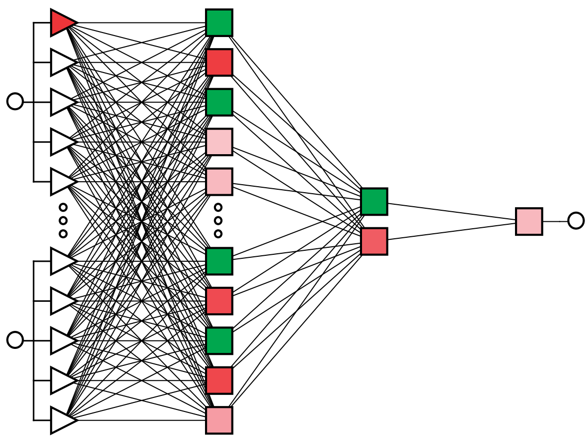

Fig. 11. An architecture of the neural network MLP 5:42–12–2-1:1 for withdrawals forecasting

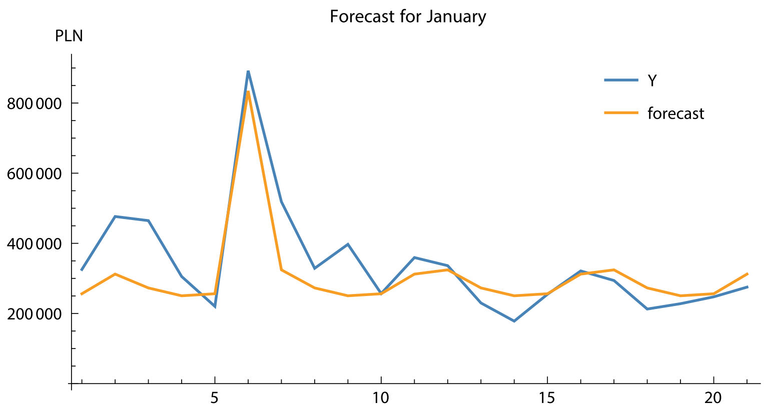

Fig. 12. Time series of actual (Y) and forecasted withdrawals obtained by machine learning model

3 Discussion

The methodology of model creation and assessment criteria are significantly different for the models considered in the paper (i.e., for ARMAX and MLP). However, comparison of forecasts generated by both models seems to be useful. The Mean Absolute Percentage Error was assumed as a criterion for the comparison. Table 11 includes values of MAPE for each testing set.

The forecasts obtained for a testing subset T1 are very similar and satisfactory in terms of the goal of forecasting. In the longer period of forecasts a clear advantage of the ARMAX model is encountered. However, we should consider whether increasing error of the MPL model MLP is natural and expected. The approach with the window shifted in time should minimize this drawback. Undoubtedly, distinctiveness of the considered models in terms of results of forecasts can be utilized in the hybrid model.

Tab. 11. Values of Mean Absolute Percentage Error for both models for each testing set

	Sub-period				
Model	**T1**	**T2**	**T3**	**T4**	**T1+T2+T3+T4**
MLP 5:42-12-2-1:1	18,47	25,61	24,42	45,81	31,03
Model ARMAX	19,09	19,14	24,19	12,87	18,81

Conclusions

Our studies and analyses have shown that the forecasting of customers withdrawals in the banks is a complex and difficult task. In our experiments, the major constraint was the restriction of the time series of withdrawals to 18 months. It significantly reduced the possibility of capturing important calendar and seasonal characteristics. Variables and attributes reflecting the calendar effects, used in both models, in case of obtained errors of forecasts less than 20%, showed a significant, non-linear influence of this type of predictors on the amount of the daily cash withdrawals in the bank, and hence on the amount of the daily declared cash limit.

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