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A Proposition for a Methodology to Assess the Influence of European Union Funds on Living Conditions among Citizens of a Commune

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Abstract

In this paper a two phase methodology to assess the influence of the European Union funds on living conditions among citizens of a commune was proposed. During the first the total index of living conditions in a given commune during a specific time period is calculated. During the second phase econometric logit models of the dependence of the total index on the share of the EU funds invested in a given commune are constructed. The developed methodology has been illustrated with calculations concerning the small, rural commune Zarzecze from the Podkarpackie Voivodship during the years 2003–2013.

Keywords: total and partial indexes of living conditions, European Union funds, econometric models of living conditions

Introduction

Living conditions are a set of objective factors influencing the level of satisfaction in material needs related to the income situation of households, the job market, housing conditions and equipment of households, health services, education, culture and social assistance (Czapiński and Panek 2013; Ostasiewicz 2002). Similar to the reality surrounding a human being, living conditions are complex and multidimensional. In order to assess their changes one needs to take a special approach — multidimensional comparative analysis. This consists in transformation of the ratings of the multidimensional, social and economic reality surrounding the human being into a one-dimensional complete index of living conditions. Most often this kind of research applies to whole countries or regions. It is much more rare to perform research on changes of living conditions of citizens of communes, especially lesser rural ones, due to their small area and lack of some statistical data. It does not mean that this problem is not important — society consists of small communes. Learning the mechanisms of changes in living conditions of small societies may provide a great deal of substantial information related to similar changes in larger areas.

Living conditions are determined by numerous factors — economic, social and political. One factor which appeared in Poland in the beginning of the 21st century, is European Union funding. This is allocated into both large, infrastructural projects influencing the development of the whole country and small developments implemented by local societies. This would mean that the EU funds should improve the quality of living conditions. However, due to the multidimensional meaning of this notion and the inconstant number and value of EU co-financed projects over time it is essential to create a special methodology in order to assess the influence of the funds on the commune level.

The proposed methodology consists of two phases. During the first the total index of living conditions in a given commune during a certain time period is calculated. During the second phase econometric logit models of the dependence of aggregate indicators on the share of the EU funds invested in a given commune are constructed. The developed methodology has been illustrated with calculations concerning the small, rural commune Zarzecze from the Podkarpackie Voivodship during the years 2003–2013.

1 The methodology of studying the influence of aid funds on the living conditions of the citizens in a commune

The hypothesis that EU funds do improve living conditions of citizens in a commune has been stated with the years of 2003–2013 as the timeframe of the study. This means that the following were included:

- the year 2003, which is the period just before joining the EU, as a reference point
- years 2004–2013 in an EU commune, using aid funds

The basic hypothesis has been verified via means of econometric models, whose dependent variable was the total index of living conditions of the commune's citizens. The independent variables were those describing the amount of EU fund commitment. The proposed methodology consists of the following phases.

1.1 Calculating the value of the total index of living conditions during the analysed timeframe.

1.1.1 Separation of sectors describing the living conditions

The citizens' living conditions is a multidimensional category, which shall be described by means of r components, referring to individual sectors of living conditions. Each of these sectors shall be described with a set of variables $[x_{ijt}]$, where x_{ijt} is the value of the i-th variable, which is included in the j-th component (sector) in the t year.

1.1.2 Transformation of the potential variables explaining individual sectors into indicators

Each one of the specified variables has an appropriate name (%, pcs., km, PLN, etc.) and a very diverse range of variability. This means that they cannot be compared directly with each other and cannot be summed in order to acquire the total values. Additionally, we may deal with three types of variables: stimuli, when the higher variable amount means better living conditions, destimuli, when the higher amount means worse living conditions and neutral variables (nominants). In the last case, the best living conditions exist when this variable has a fixed value — lower and higher ones mean worse conditions (this type of variable appears very rarely).[1] In order to be able to add values of different variables describing various, sometimes very dissimilar components of life conditions, a process called unitarization needs to be applied: transformation into the values ranging from 0 to 1. In this work, the following procedure of unitarization has been applied (Kukuła 2000, 79):

(1) $$\text{for stimuli variables:} \quad z_{ijt} = \frac{x_{ijt} - x_{ij\,min}}{x_{ij\,max} - x_{ij\,min}},$$

(2) $$\text{for destimuli variables:} \quad z_{ijt} = \frac{x_{ij\,max} - x_{ijt}}{x_{ij\,max} - x_{ij\,min}},$$

where:

x_{ijt} — the value of the i-th variable in the j-th life sector in the year t,
z_{ijt} — the value of the i-th indicator in the j-th life sector in the year t,
x_{ijmax} — the maximum value of the i-th variable in the j-th life sector during the timeframe of the study,
x_{ijmin} — the minimum value of the i-th variable in the j-th life sector during the timeframe of the study.

1. These three types of variables are described in the work of Kukuła (2000, 53–54).

In this way potential explanatory indicators are created. They are normalized within the range [0; 1] and are stimuli, which makes it possible to compare phenomena described by means of different measures. These indicators become a subject of further calculations.

1.1.3 Selecting diagnostic indicators describing certain life sectors

Potential explanatory variables which are a part of certain sectors (components) may be heavily correlated with each other, which distorts the values of total measures.[2] This is the reason why the formal variable selection method has been chosen. It is based on a condition number of a correlation matrix of potential indicators describing each R_l sector (Malina and Zeliaś 1996, 86). The selection process for an optimal set of indicators describing the l-th life sector proceeds as follows:

- Calculating the correlation matrix $\mathbf{R} = [r_{ij}]$ between potential explanatory indicators.
- Determining the matrixes $\mathbf{R}^{-1} = [r_{ij}^{-1}]$. It is worth noting that the diagonal element r_{ij}^{-1} of the $\mathbf{R}^{-1}$ matrix equals unity, if the Z_{ij} indicator is orthogonal in relation to remaining indicators. If it is not orthogonal, then $r_{ij}^{-1} \in (1; \infty)$. When the indicators are too heavily correlated with each other, the diagonal elements of a reverse matrix $\mathbf{R}^{-1}$ are much larger than unity, which is a symptom of an ill conditioned matrix $\mathbf{R}$.([3])
- Eliminating indicators controlled by means of the measure of ill conditioned matrix (Položij 1966, 349). Let us take a number N to become the measure of an ill conditioned $\mathbf{R}$ matrix:

  $$N = \frac{1}{n} N(\mathbf{R}) N(\mathbf{R}^{-1}). \tag{3}$$

  The smaller the N number is, the better the matrix numerically conditioned becomes. $N(\mathbf{R})$ is defined as a root mean square of the elements forming the $\mathbf{R}$ matrix, becoming its norm:

  $$N(\mathbf{R}) = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} r_{ij}^2}. \tag{4}$$

  The best conditioned matrixes are the orthogonal ones. In these the number conditioning the matrix equals unity. This value should be strived for, by eliminating in sequence those indicators which correspond with the largest values of the diagonal elements of the $\mathbf{R}^{-1} = [r_{ij}^{-1}]$ matrix.
- In practice, it is difficult to attain an orthogonal matrix with more than one indicator. This is why a so-called "stop algorithm" $N(\mathbf{R})$ is subjectively established. The researcher determines it is small enough and after exceeding it calculations are stopped.

Diagnostic indicators chosen by means of this method for each life sector become the basis for calculating partial (concerning separate sectors), and then the total index of the commune's living conditions.

1.1.4 Calculating the partial and total index of living conditions

Partial and total index of living conditions are formed as a function of selected diagnostic indicators. In the literature, there are many proposals for such indexes. The first suggestion was created by prof. Zdzisław Hellwig (1968). The total index of development he proposed became an inspiration for several generations of researchers who developed the original idea. It is worth mentioning the works of Pociecha, Podolec, Sokołowski and Zając (1988), Grabiński, Wydymus and Zeliaś (1989), Nowak (1990), Kowerski (1983), Jajuga (1993), Pluta (1977), Zeliaś (2000), Kukuła (2000) and Panek (2009).

Analysis of the earlier accomplishments leads to a proposal of the following procedure of calculating the partial indexes of commune citizens' living conditions over the analyzed timeframe

2. Introducing variables heavily correlated with each other causes the total index to have its distribution similar to the distribution of these correlated variables. Heavy correlation means that the variables have very similar information about the studied phenomena. In such a case the total index distorts correct results.

3. A square matrix $\mathbf{R}$ is ill-conditioned if a reverse matrix is unstable (i.e., small changes in elements of one matrix cause big changes in the elements of the reverse matrix).

(5) $$M_{jt} = \frac{1}{k}\sum_{i=1}^{k} z_{ijt},$$

where:
M_{jt} — partial index of the j-th sector of living conditions in the year t,
k — amount of diagnostic indicators of the j-th sector of living conditions,
z_{ijt} — the value of the i-th indicator of the diagnostic indicator of the j-th sector of conditions in the year t.

Total index of the commune citizens' living conditions is the arithmetic average of partial indexes

(6) $$M_t = \frac{1}{r}\sum_{j=1}^{r} M_{jt}.$$

Both, partial and total index, take value within the range [0; 1].

1.2. Building econometric models of total living conditions index

1.2.1. Specification of the model

As the dependent variable is a proper fraction, a logit transformation has been applied.[4] This eliminates a situation where estimated theoretical values exceed the [0; 1] range.[5] Logit transformation is converting respective empirical probabilities (standardised indexes of living conditions) into logits, according to the formula (Gruszczyński 2001, 19)

(7) $$\text{Logit} = \ln\frac{M}{1-M},$$

where M is the total index of living conditions within normalized the range [0; 1]. Instead of index of living conditions specified in the range [0; 1], a value of directly corresponding logit value is achieved. The next step in building models of variable normalized by means of logits is the estimation of relations between empirical values of logits and independent variables taken into account. Hence, estimated models of the total index of living conditions appear in this form

(8) $$\text{Logit}M_t = \ln\frac{M_t}{1-M_t} = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + \alpha_4 \text{CEU}_t + \varepsilon_t,$$

where CEU_t is the percentage of cumulative (since 2003)[6] EU funds in total cumulative expenses of the commune in the year t.

A hypothesis stating that the living conditions in a commune were influenced by the economic situation in Poland shall also be verified. In order to achieve this, the following model shall be used:

(9) $$\text{Logit}M_t = \ln\frac{M_t}{1-M_t} = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + \alpha_4 X_t + \alpha_5 \text{PKB}_t + \varepsilon_t,$$

where PKB_t is the GDP growth rate of Poland in the year t. If the results of estimations will be of low quality, autoregressive models shall also be considered

(10) $$\text{Logit}M_t = \ln\frac{M_t}{1-M_t} = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + \alpha_4 \text{PKB}_t + \alpha_5 M_{it-1} + \varepsilon_t.$$

Introducing a polynomial trend of degree 3 stems from the assumption that during the analyzed timeframe the changes of living conditions may not have a linear characteristic (which is

4. It is a monotonic transformation of probabilities (normalised indexes of living conditions) from the range [0; 1] to $(-\infty; \infty)$. This allows the dependent variable to avoid having a finite interval. The concept of logit transformation was proposed by Ronald A. Fischer and Frank Yates in 1938. In 1944 Joseph Berkson, a physicist and a statistician introduced the term logit (Agresti 1996, 261–262).

5. An exemplification of the problem, see Goldberger (1972, 320).

6. For the year 2003 cumulative funds are equal to the funds spent in 2003. For 2004, cumulative funds are equal to the sum of the funds spent in 2003 and 2004 etc.

most commonly used in research), but might be influenced by non-linear fluctuations. However, the final choice of models of total index of living conditions shall be carried out by means of the a posteriori elimination method (Nowak 1998, 139–142). The models with the highest value of the determination coefficient, meeting all the hypotheses concerned with the formation of random elements, shall be chosen.

Estimating parameters of logit models has been performed through the method of least squares. If the assumption of the lack of autocorrelation or homoscedasticity of random elements had not been met, the heteroskedasticity corrected method or Cochrane-Orcutt procedure, with using the GRETL programme (Kufel 2011), have been employed. The assessment of the measure of goodness of fit of the model with the empirical data has been performed by using the determination coefficient R^2. Because of the small amount of observations and small number of degrees of freedom, the additional step was to calculate the adjusted determination coefficient $\bar{R}^2$. By using the F Fisher-Snedecor test, it was also possible to study the significance of the multiple correlation coefficient, measuring the significance of dependency between the dependent variable and all the independent variables appearing in a given model. Coincidence of the estimated parameters has also been taken into account (Hellwig 1976).

1.2.2 The interpretation of the logit model

A logit model is a linear model of a logit of dependent variable relative to the independent variables. This is why individual parameters are derivatives of logits relative to corresponding independent variables. Their assessments are interpreted in the same way as in the case of a linear econometric model. Changes of the logit itself are, however, not really useful for interpreting changes of a studied phenomenon. Hence, it is much more popular to employ interpretation related directly to a predicted (theoretical) level of frequency, calculated basing on the estimated model (Gruszczyński 2001, 59). Basing on the logit model, it is possible to calculate the theoretical values of living conditions index

$$\hat{m}_t = \frac{e^{\hat{y}_t}}{1 + e^{\hat{y}_t}}, \tag{11}$$

where $\hat{y}_t$ the estimated value of logit in the year t.

By using the equation above it is possible to study changes in both partial and total indexes of living conditions in relation to the share of the EU funds in the commune's expenditure. This function has a logistic distribution, and its variability may by calculated by analytic means. If—according to the accepted hypothesis—the value of the α_1 parameter is positive, it means that the independent variable is a stimuli. In such case, growth of value of living conditions' index increase to a point of inflection (acceleration phase). After crossing the inflection point the growth of value become smaller (deceleration phase).

While the direction of the probability change (derivative sign) of the indicator is specified only by the α_1 sign, the amount of the change in the indicator value in the year t (if the independent variable changes by a unit) depends on the value of the independent variables vector. This means that it changes along with the changes of the independent variables and thus the derivative is most commonly calculated for the medium (average, median) values of the independent variables within a model.

2. The influence of the European Union on changes in living conditions of the citizens of Zarzecze Commune—the results of the research

2.1. A brief description of the Zarzecze Commune

The commune of Zarzecze is located within the Rzeszów Foothills, in the central part of the Podkarpackie Voivodship. It is a part of the county of Przeworsk. It is a rural commune, with an area of 49,24 km^2. It consists of 9 settlements forming 10 village councils. In the end of 2013 the population of the commune was ca. 7 200. The population density is 146 people per km^2 and is higher both than the average for the Podkarpackie Voivodship (102 per km^2) and the average for the whole country (123 per km^2) (*Powierzchnia i ludność...* 2013). In the end of 2013 there were 339 economic entities registered within the commune, with a domination of small businesses (either self-employment or small companies hiring a couple of persons). The amount of entities has significantly grown since 2002. The biggest growth has been noted since 2009. Most of the businesses operating within the commune deal with wholesale and retail trade and car repair services (64 companies) or construction (22 companies).[7] It is an agricultural commune with a good quality of soil (Strategy, 2007). Most commonly grown crops are grain, sugar beets and potatoes. Most common animal types are cattle and swine. 94% of citizens use the water supply network. Its length in the range of the commune is 71,1 kilometers and 1737 households are connected. The water supply network and the water treatment station were modernized in the years 2008–2010. The sewage network has a length of 50,6 kilometers and is used by 82,2% of citizens. In the commune there is also a modernized sewage treatment plant. Every settlement has access to the national gas network. Its length is 102 kilometers and it is used by 75,9% of citizens.[8]

2.2 Use of European Union funds in the years 2003–2014

During the studied timeframe, the commune of Zarzecze carried out 22 projects with a total cost of PLN 34,7 million, 78% of this value was financed from the EU funds. Sixteen projects co-financed via SAPARD, PROW and Podkarpackie Voivodship RPO projects were of a modernization and investment nature (building roads and pavements, water pipes, modernizing the Communal Centre of Sports and Recreation, modernizing and building educational and sports facilities, fire stations, renovations of historic buildings, modernizing settlements). In total these projects cost PLN 23,2 million and 67,3% of this value was financed by the EU funds. The other six projects, of PLN 11,5 million value were related to the development of human capital (labor market re-entry, improving the quality of education). These were financed 99,5% by the EU. During the analyzed timeframe the commune received EU funds each year, except for 2006. The biggest share of EU subsidies in total amount of expenses took place in 2010 (23,1%).

Tab. 1. Structure of projects co-financed by EU funds carried out in the commune of Zarzecze in the years 2000–2013

Type of project	Number of projects	Total value of projects (thousands of PLN)	Amount of subsidies (thousands of PLN)	Share of subsidies in the total value of projects (%)
Investment and repairs	16	23 175	15 603,5	67,3
Human capital	6	11 503	11 449,0	99,5
Total	22	34 678	27 052,5	78,0

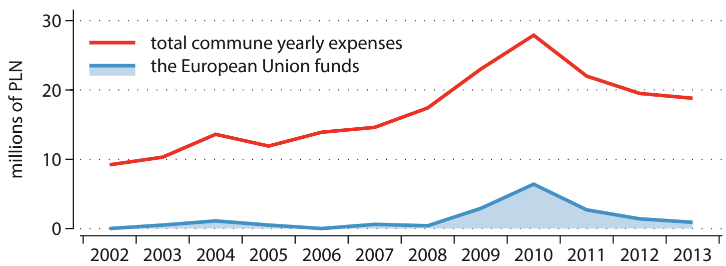

Fig. 1. EU funds and total commune expenses in the years 2002–2013

2.3 Variables describing living conditions of the citizens in Zarzecze Commune

Analysing social processes and the possibility to access data made it possible to isolate 7 sectors of conditions explained by 19 potential independent variables (see tab. 2 and fig. 2).

7. The registry of businesses in the Commune Office of Zarzecze.

8. [In the journal European practice of number notation is followed — for example, 36 333,33 (European style) = 36 333.33 (Canadian style) = 36,333.33 (US and British style). — Ed.]

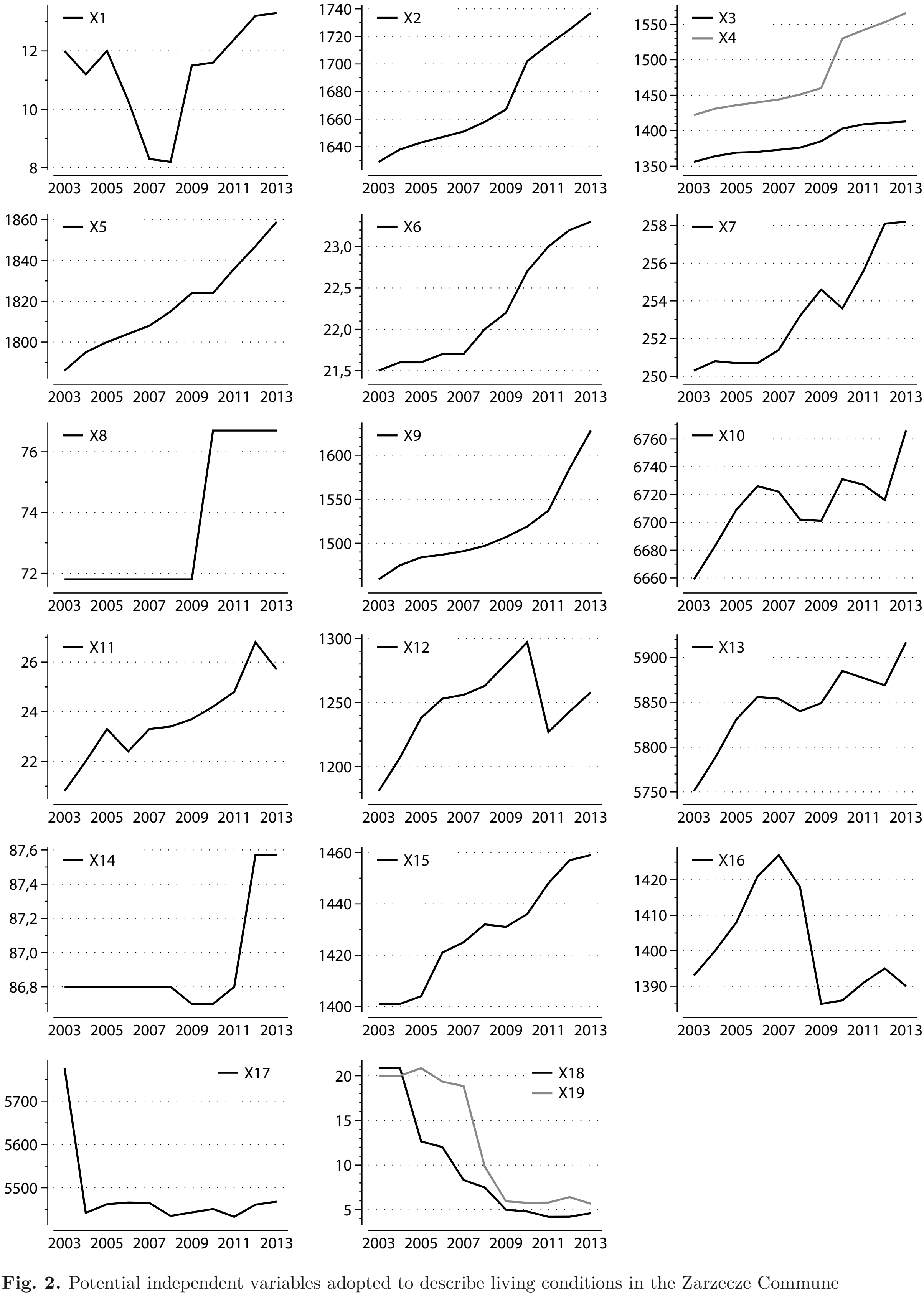

Fig. 2. Potential independent variables adopted to describe living conditions in the Zarzecze Commune (symbols of variables are described in table 2 on next page)

Tab. 2. Potential independent variables adopted to describe living conditions in the Zarzecze Commune

Sector of living conditions	Measure unit	Variable symbol
Job market		
• the number of registered unemployed persons in relation to the number of persons of working age	%	X_1
Technical and sanitary utilities in households		
• water network supply	pcs.	X_2
• gas from network	pcs.	X_3
• bathroom	pcs.	X_4
Housing resources		
• housing resources in total	pcs.	X_5
• average usable dwelling area per person	m^2	X_6
• number of dwellings per 1000 citizens	pcs.	X_7
Water supply network		
• length of the active distribution network	km	X_8
• connected houses and blocks of flats	pcs.	X_9
• number of people using the water supply network	person	X_{10}
• water consumption per person	m^3	X_{11}
Sewage system		
• connected houses and blocks of flats	pcs.	X_{12}
• number of people using the sewage network	person	X_{13}
Gas network		
• total length of the active network	km	X_{14}
• active connections with dwellings	pcs.	X_{15}
• gas recipients	household	X_{16}
• number of people using the gas network	person	X_{17}
School computerisation. Number of students per 1 computer with Internet access, for students' use		
• primary schools	pcs.	X_{18}
• middle schools	pcs.	X_{19}

2.4 Result of selecting the diagnostic indicators

Every potential independent variable underwent the procedure of normalization. Then, for each sector an elimination of indicators was performed in order to achieve diagnostic indicators. The job market sector did not have to have its indicators eliminated, as it possessed only one, which automatically became a diagnostic indicator. In the rest of the cases it was decided that the procedure should be continued until the norm becomes smaller than 4. Twelve diagnostic indicators have been chosen.

Tab. 3. Result of selecting diagnostic variables by the condition number of a correlation matrix method

Sector of living conditions	Value of the N norm	Chosen diagnostic variables
Job market	x	Z_1
Technical and sanitary utilities and furnishings in households	94,97	Z_3
Housing resources	38,91	Z_7
Water supply network	2,51	Z_8, Z_{10}
Sewage system network	3,35	Z_{12}, Z_{13}
Gas network	2,95	Z_{14}, Z_{15}, Z_{16}, Z_{17}
School computerisation	5,78	Z_{19}

2.5 Total index of living conditions in the Zarzecze Commune in the years 2003–2013

During the analyzed timeframe the living conditions of the citizens in the commune of Zarzecze grew, apart from a small decline in 2009. The total index of living conditions rose from 0,13 in 2003 to 0,74 in 2013. Still, it is worth mentioning that if the values of diagnostic indicators had grown systematically, then (according to the accepted methodology) the total index would have achieved the value of 1 in 2013. This did not happen. From the perspective of 2013 it can be said that in some sectors of life "earlier was already better." However, the basic conclusion does not change: living conditions in the commune improved.

2.6 Results of estimations of models describing the influence of European Union funds on the living conditions of the commune of Zarzecze

The obtained total indexes in respective years were transformed into logits. Then, models were built, conditioning their value on percentage share of cumulative EU funds in cumulative commune expenditure and other variables.

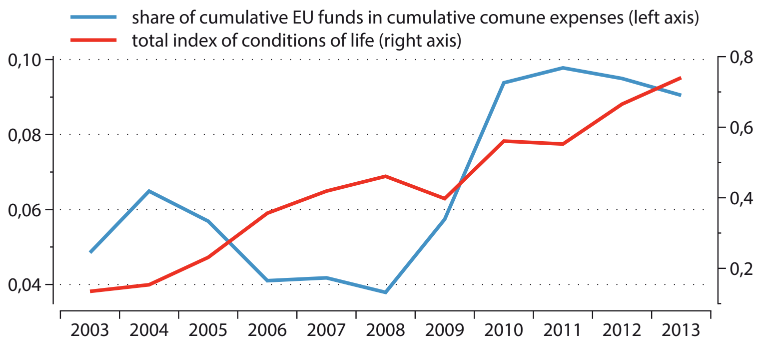

Fig. 3. Share of cumulative EU funds in cumulative commune expenses and total index of living conditions of Zarzecze in the years 2003–2013

The estimated polynomial trend of degree 3, "enriched" by the GDP growth rate is a good image of temporary worsening of living conditions in 2009. It also shows that the direction of the changes in living conditions was similar to the changes of the GDP of Poland. Positive and statistically significant values of parameters of the variable describing the percentage share of cumulative EU funds in cumulative expenses of the commune (CEU_t) — models 2 and 3 — confirm the positive influence of the EU funds on the conditions of life within the commune. However, a relatively low level of model fitness to the empirical data, and – in the case of a model estimated by ordinary squares method — the autocorrelation of random components makes it impossible to acknowledge the EU funds as the only mean of the improvement of living conditions. The level of fitness greatly enhances the autoregressive model (4), where the dependent variable delayed in time may be a proxy of other no-EU fund factors (although it does not explain which ones) determining the advance living conditions. Great growth (in comparison with models 2 and 3) of coefficient R^2 supports the notion that the EU funds were not even the most important source of the improvement

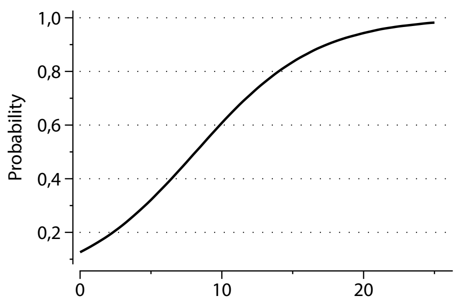

Fig. 4. Influence of share of cumulative EU funds in cumulative commune expenses (%) on the total index of living conditions. Estimations on model 2

Tab. 4. The results of estimation of models of Total index of living conditions (Logit M_t) in Zarzecze Commune in the years 2003–2013

Specification	Model 1 (OLS)		Model 2 (OLS)		Model 3 (Heteroskedasticity corrected)		Model 4 (Cochrane-Orcutt)	
	Coefficient	***p*-value**	**Coefficient**	***p*-value**	**Coefficient**	***p*-value**	**Coefficient**	***p*-value**
Constant	−2,551	< 0,0001	−1,934	0,0290	−1,863	0,0451		
t^2	−0,089	0,0219						
t^3	0,006	0,0227						
PKB_t	0,116	0,0011						
CEU_t			0,237	0,0258	0,242	0,0256		
CEU_{t-1}							0,038	0,0038
Logit M_{t-1}							0,850	< 0,0001
R-squared	0,9560		0,3701		0,4420		0,9018	
Adjusted R-squared	0,9366		0,3001		0,3800		0,8878	
	Statistic value	***p*-value**	**Statistic value**	***p*-value**	**Statistic value**	***p*-value**	**Statistic value**	***p*-value**
F test Null hypothesis: Multiply correlation coefficient is not significant Statistic F	138,87	< 0,0001	7,11	0,0258	7,13	0,0256	83,16	< 0,0001
Test for normality of residual Null hypothesis: disturbance term is normally distributed Test statistic	0,10	0,9521	1,54	0,4624	1,47	0,4806	1,74	0,4197
White test for heteroskedasticity Null hypothesis: heteroskedasticity not present Asymptotic test statistic z	8,75	0,4611	2,76	0,2519				
Durbin-Watson test Null hypothesis: no autocorrelation d statistic	2,64	0,5255	0,5	0,0001				
Test for ARCH of order 1 Null hypothesis: no ARCH effect is present Test statistic: LM	0,92	0,3385	1,69	0,1935	2,78	0,0954	0,41	0,5204
CUSUM test for parameter stability Null hypothesis: no change in parameters Test statistic: Harvey-Collier	0,86	0,4209	6,23	0,0003				

of living conditions. It is also worth mentioning that in the case of model 4 the variable describing the share of EU funds is delayed by 1 year, which means that these spent funds affect living conditions with a delay (see tab. 4 on previous page)

Conclusion

The proposed methodology makes it possible to assess the changes in the living conditions of a commune's citizens and define the influence of the EU funds on these alterations. Because of referencing the values of respective variables to their maximums and minimums during the analysed timeframe it is recommended to repeat the calculations every year, as more up-to-date data become available.

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